Lecture 3—Friction

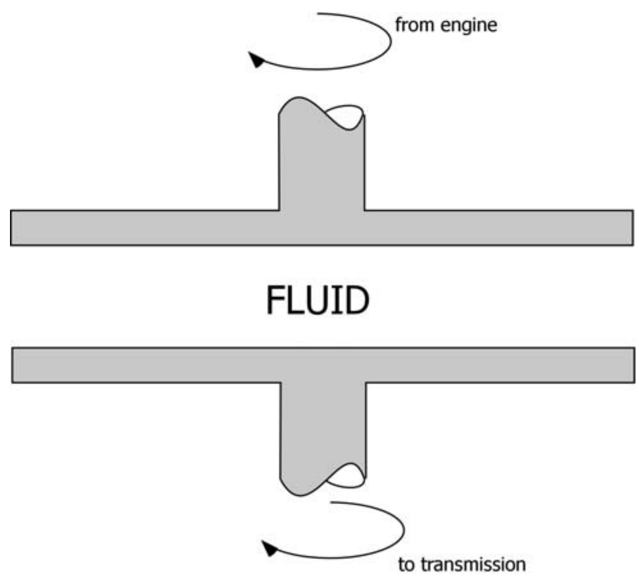
Last time we ended by noting that the Energy Grade Line isn't really horizontal—there's a head loss, h_l, that causes a slight negative slope to the EGL. What causes head loss? Really, head loss is just another form of energy (heat), but because this energy cannot be converted to any other form of energy, it is considered "lost" to the system, and isn't accounted for in the Bernoulli equation. This becomes of vital importance to us because up to now, we've been talking about water as though all the water in a pipe or a river flows at the same speed, and we *also* know (from our buddy Bernoulli, again) that speed *differences* in fluids give rise to *lift*, and THAT is what's going to move sediment. The energy loss we're talking about results from *shear* within the fluid, which implies that the fluid moves at different speeds at different points in the flow.

The best way to start talking about this frictional loss is somewhat tangentially. Does anybody know how a manual clutch works? All cars have a problem—the engine and the transmission can't turn at the same speed all the time, because if they were forced to, the engine would stop running when the car wasn't moving (try it with your standard transmission!). In a standard transmission, this problem is solved by putting a plate in between the engine and the transmission, and connecting the plate to a pedal. When you push the pedal, the plate disengages the engine and the transmission, and the engine is free to continue turning while the transmission doesn't.

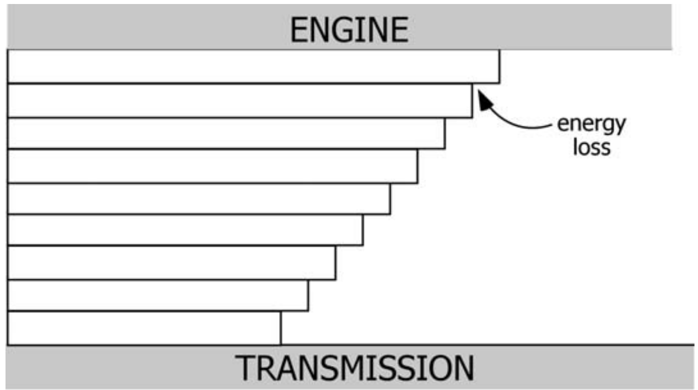
Now, surely you all know *someone* who just doesn't seem to get the whole shifting thing in a standard transmission. The car manufacturers of America knew someone, too. And they figured out that if they could make a car that not only *shifted itself*, but also didn't need to have a pedal pushed in just to get the car to stop, *they'd sell more cars*. Enter the automatic transmission.

So how does an automatic transmission achieve the odd ability to get the car to stop but not the engine? It happens through a fascinating device called a *torque converter*. {see ppt}. A torque converter takes the place of a clutch in a manual transmission—basically, the engine spins a plate that forces *fluid* (automatic transmission fluid) to spin, and *that* induces spin on the transmission flywheel. That's it! At low engine speeds (idle), the fluid only induces a little movement in the transmission, so you only have to have light brake pressure to keep the car from moving.

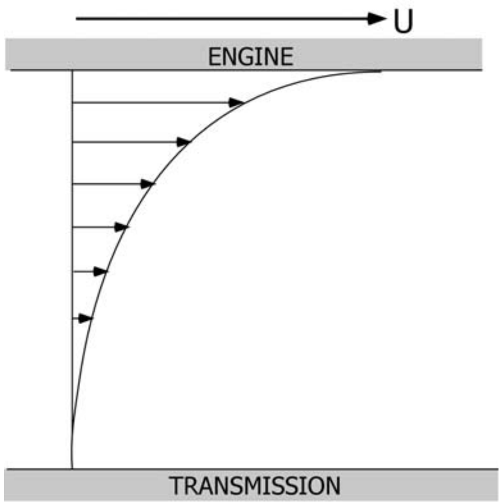
Ever wonder why automatic transmissions get worse fuel economy than standards? It's because of the torque converter! Just like in water flowing through a pipe, some of the energy transmitted from the engine is lost as heat before it ever gets to the transmission. To understand why, let's take a closeup look at a (simplified) torque converter:

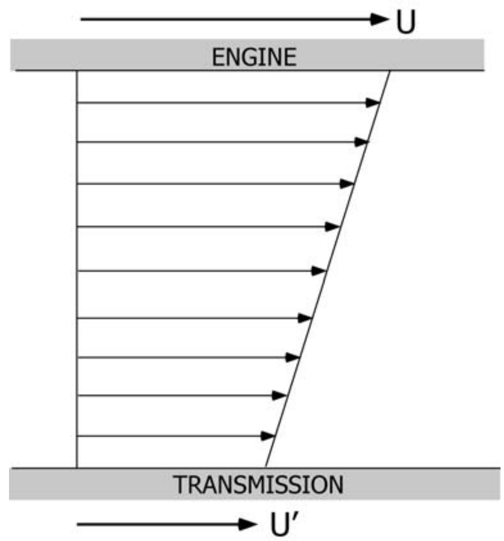


What's happening is that as molecules of transmission fluid slide along the bottom, or even slide past each other, some of their energy of motion is dissipated as heat. Picture the fluid gap as very very thin slices of fluid:



And we're going to start moving the flywheel. What happens?





First the plate induces the uppermost layer of fluid to move at the plate's velocity (this is called a "no-slip" boundary). But, within the fluid, each little layer of fluid under the uppermost layer moves at a *slightly* slower velocity, because part of the energy has been dissipated as heat. If you keep on with the plate, however, you eventually reach a point where the engine flywheel turns at a *slightly* higher velocity than the transmission flywheel, and that the difference is proportional to the speed to the engine flywheel. Effectively, there's a velocity *gradient* in the fluid, which would be:

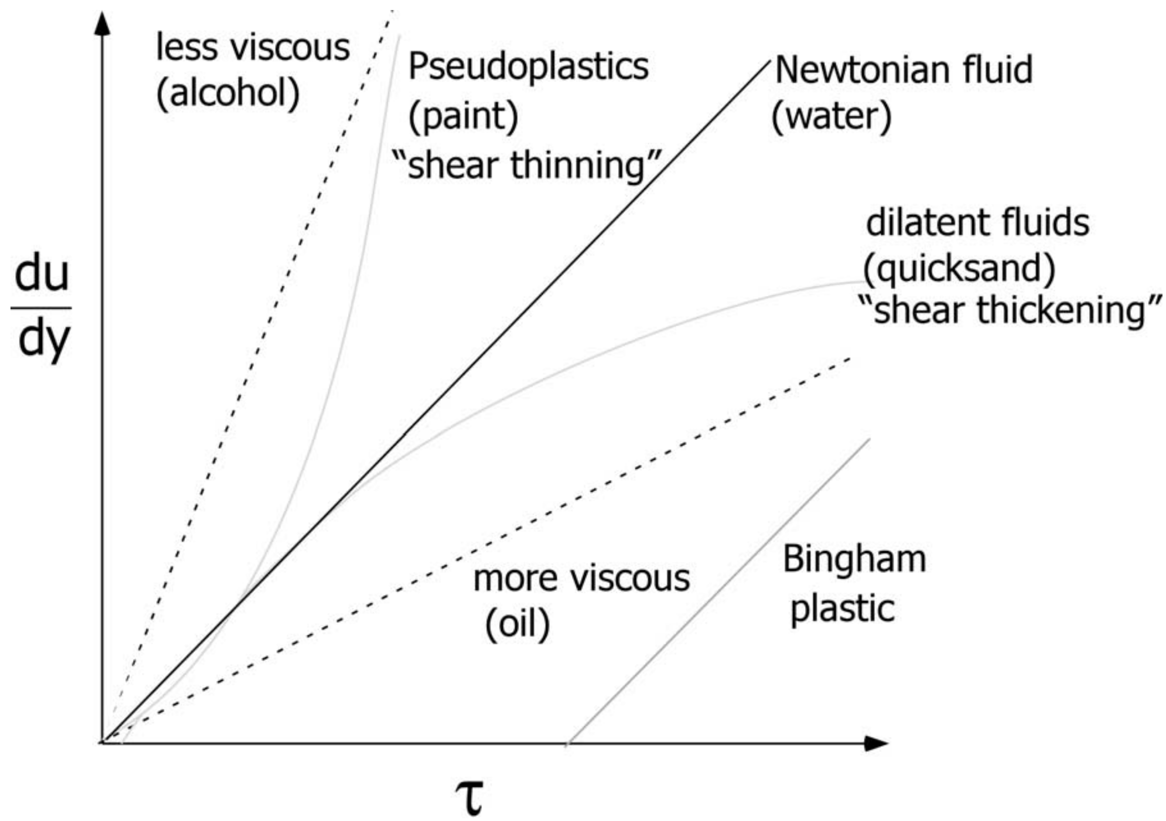
$$\frac{\textit{engine speed} - \textit{transmission speed}}{\textit{gap thickness}} = \frac{\Delta V}{\Delta L} = \frac{du}{dy}$$

And the speed of the engine flywheel is a function of how much force you put on it, right? Well, sort of—you know that it will take more force to move a bigger flywheel, so maybe we should normalize this by dividing the force by the area of the flywheel—this produces a *shear stress*, which we'll abbreviate as τ. At the moment, we have:

$$\tau(y) \propto \frac{du}{dy}$$

where the notation τ(y) just says that the shear stress varies within the fluid, so τ(0) would be the shear stress between the bottom plate and the fluid, τ(d) would be the shear stress between the top plate and the fluid, and any other τ(y) would be the shear stress between two layers of fluid located y units up from the bottom plate. Simple! Because there's a linear relationship between *velocity* and distance, the derivative is constant, therefore the shear stress is constant. We'll talk more about what happens when the shear stress isn't constant later.

You can imagine that some fluids are very effective at transmitting velocity, and others are highly inefficient. Moreover, at least for transmission fluid, *any* force on that upper plate will result in shear within the fluid. In the ideal case, there's a direct relationship between the stress acting on the fluid, and how it transmits shear through the fluid. Graphically,



For a Newtonian fluid, the slope of the line is a measure of how efficient the fluid is at transmitting velocity. The flatter the slope (on these axes), the more efficient the fluid is at transmitting velocity downward.

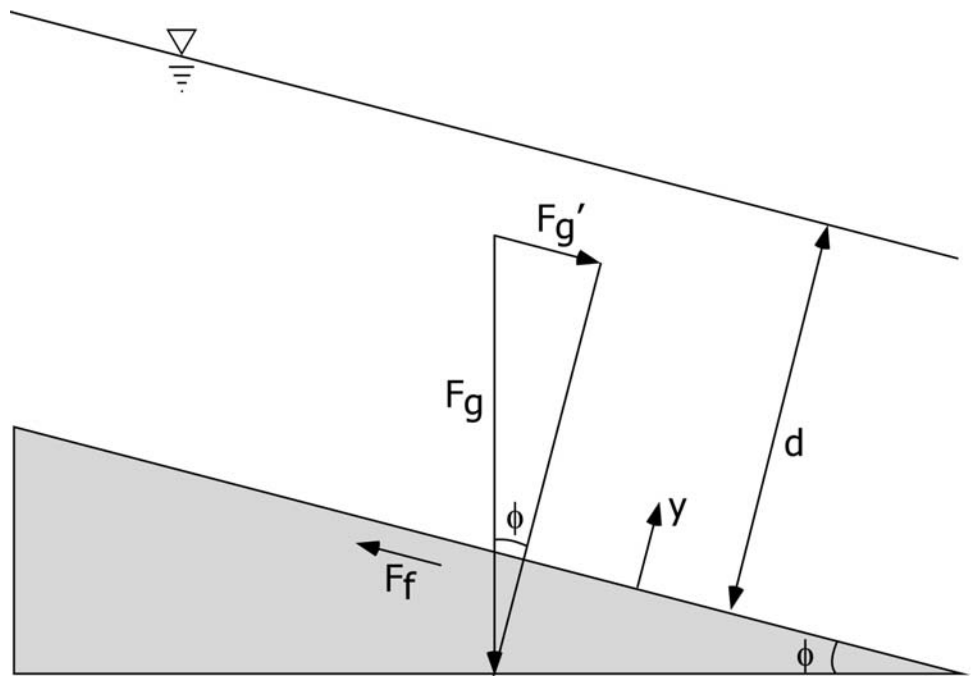
Every fluid has its own slope on this graph. Some fluids (like water) change their slope depending on the temperature. Some (like quicksand) change their slope depending on the rate of shear. But the slope (which is a way of talking about the efficiency of the fluid) is a physical property of the fluid, just like density. We call intrinsic properties like these *material properties*, and the material property of a fluid that says how efficient it is at transmitting velocity is called *viscosity*.

In math:

$$\frac{\tau}{du/dy} = \mu \text{ or } \tau = \mu \frac{du}{dy}$$

and the bigger μ is, the better the fluid is at transmitting velocity.

OK, so what does *any* of this have to do with water flowing downhill?



This is just like the torque converter, except that there's no plate on top! No plate means no stress, so at the top of the flow τ=0. At the *base* of the flow, though, the stress is exactly balanced by the force of gravity acting downslope. The gravity part is easy:

$$F_g' = \rho g d \sin\phi$$

so

$$\tau_0 = \rho g d \sin\phi$$

From here, we know that:

$$\tau = \tau_0\left(1-\frac{y}{d}\right)$$

which is just the equation of a line going from $\tau=\tau_0$ at y=0 to τ=0 at y=d.

From before, we said that $\tau = \mu\frac{du}{dy}$, and $\tau_0 = \rho g d \sin\phi$, so:

$$\mu\frac{du}{dy} = \rho g d \sin\phi\left(1-\frac{y}{d}\right)$$

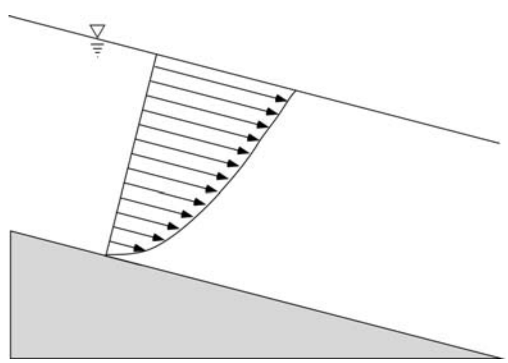
so

$$\frac{du}{dy} = \frac{\rho g \sin\phi}{\mu}(d-y)$$

and integrating:

$$u = \frac{\rho g \sin\phi}{\mu}\left(yd - \frac{y^2}{2}\right)$$

which is the equation of a parabola:



This is what happens if viscous forces are the only ones involved in the problem; it works for a relatively viscous fluid (like ice, or transmission fluid) on a perfectly smooth bottom. For a rough bottom, however, we get flow separation and non-parallel stream lines, so:

$$\tau = \mu \frac{du}{dy} + \text{turbulence}$$

And the velocity profile changes: