Lecture 7—Friction factors

We left off with a problem—we developed a velocity profile (and therefore an average velocity) for laminar flow, and one for turbulent flow, and we developed a number (Reynolds number) that supposedly tells us when to use which one, but we noted that sometimes the flow is *neither* laminar nor turbulent. Oh, crap. We also decided we didn't want to try to develop a velocity equation for a flow that's neither laminar nor turbulent. What to do?

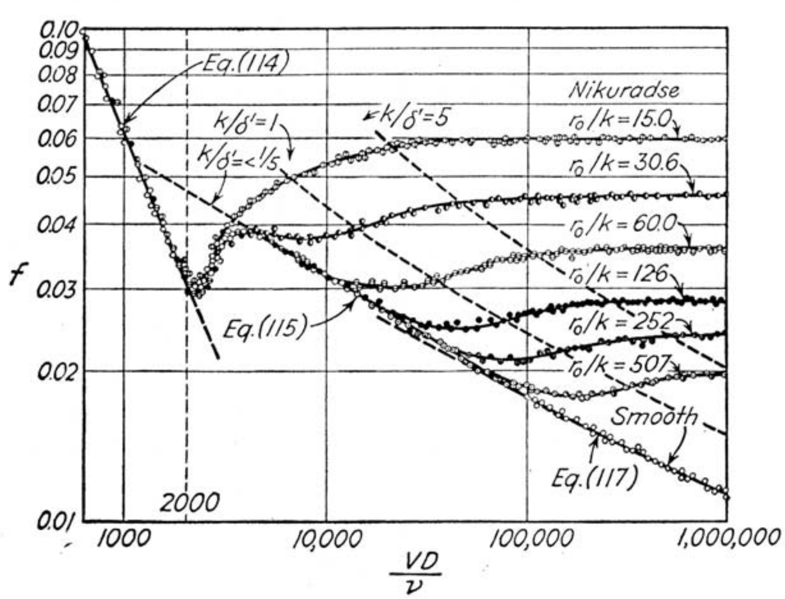
A brilliant solution comes from the heroes of the lecture—Moody and Nikuradse (and his sidekick, Brownlie). It turns out it's really hard to develop an equation based on both types of flow, so (as usual) people turn to empiricism. We already know turbulent velocity is related to the height of roughness elements on the bed, and that laminar velocity is related to viscosity (which is related to Reynolds number). SO, what if we made a chart that related Reynolds number and relative roughness to average flow velocity? That'd be really neat.

The first guy to do this was Lewis Moody, in 1944. Moody was a "sanitation engineer", and as a result, his diagram attempts to talk about how big a pump you have to put on one end of a pipe to make water flow given roughness in the pipe. This results in the first curiosity in the Moody diagram—the Reynolds number uses *pipe diameter* for the length scale, and not flow depth. This also means the *relative roughness* uses pipe diameter. When geologists, who don't care much about pipes, use this diagram we have to convert pipe diameter to flow depth. $D=4R_h$, where R_h is the hydraulic radius (and therefore about equal to the flow depth).

The Moody diagram works well, except for one issue—it was originally designed for pipes, and deals principally with rust and corrosion for the roughness elements. What if we wanted something a little more, say, *natural*? A German guy named Johann Nikuradse tried to find out more, too. Before Moody, even, Nikuradse (1933) was gluing sand to the inside of glass pipes and measuring the frictional velocity loss. Of course, Nikuradse was, well, GERMAN, so it wasn't until 1958 that Rouse brought his data to light. Curiously, even though a Nikuradse diagram appears in Rouse's 1958 book, most people ascribe the modern Nikuradse diagram to Brownlie (1983). Go figure. Remember that Nikuradse's diagram has the same issues Moody's does—you have to use

D=4R_h. Next issue—Nikuradse shows data for *uniform* grain size; when the grain size is mixed, the larger roughness elements dominate, and you end up with a different curve. So far as I know, there's not a lot of data about mixed grain sizes.

Bottom line is this—Nikuradse is probably a better choice if you're using sand, but remember that it's designed for uniform sand. Most important, note that *both* diagrams tend to have a weird "gray zone" between laminar and turbulent flow, and that's not an accident. It's because a true frictional diagram gets messy:



(Rouse, 1946)

{Practice using Moody and Nikuradse diagrams!}

This gets us a number, f, which is related to the *head loss* way back in the Bernoulli equation. Still, we'd *rather* have a velocity. How to find one? It turns out that friction factor is related to velocity through the following equation:

$$\frac{U}{u_*} = \sqrt{\frac{8}{f}}$$

So if you knew the friction factor, and you knew the shear velocity, you could determine the average flow velocity. And it turns out you'll often know the shear velocity, because:

$$u_* \equiv \sqrt{\frac{\tau_0}{\rho}} = \sqrt{gRS} \approx \sqrt{ghS}$$

Ok, so we can now get an average flow velocity based on relative roughness and Reynolds number. Whew!

Or *not*. First problem—remember how we said that relative roughness is nothing but grain roughness? No sticks, no trees, no *bedforms*, no limpets, no chutes, pools, or river meanders? That's not terribly useful for large-scale problems like river flow. Worse, we need to know flow velocity to determine Reynolds number in the first place!

Don't get me wrong—there are *plenty* of times when the Nikuradse diagram is your friend (specifically in sediment transport problems), but for determining friction factors in a river, not so much. For this, we use a much simpler equation that's *related* to the one I just gave you. Watch!

$U = \sqrt{\frac{8gRS}{f}}$ and take $C = \sqrt{\frac{8g}{f}}$, so $U = C\sqrt{RS}$. This simple equation is called the *Chezy equation*, after its discoverer. C is called the *friction coefficient* (as opposed to friction factor). NOTE that C is dimensional!!!! Basically, C is an empirical constant—rather than doing experiments on pipes, Chezy, and more importantly *Manning*, related streams to their friction coefficients just by noting similarities between streams with things like "clean straight channel, full stage, no riffs or deep pools" and "floodplain of trees, dense to cleared, with stumps". Each is assigned a range of friction coefficients. A further wrinkle here is that Manning, who did most of this, didn't use friction coefficient. Instead, he uses a factor *n*, where:

$C = \frac{1.49R^{\frac{1}{6}}}{n}$ for SAE units, and $C = \frac{R^{\frac{1}{6}}}{n}$ for MKS.

This can be combined with the Chezy equation to make the hallowed Chezy-Manning equation:

$$U = \frac{1.49}{n} R^{\frac{2}{3}} \sqrt{S}$$

Apparently we ran out of good names when it came to *n*, because it is universally referred to as “Manning’s n”. You can look up ranges of *n* in many books (including Dunne and Leopold), but *the* guide is US Geological Survey Water Supply Paper 1849, “Roughness Characteristics of Natural Channels.” It has page after page of streams and their associated Manning’s n values. You literally pick your stream, then pick your Manning’s n.

References:

Moody, L. F., 1944, Friction factors for pipe flow, Transactions of the ASME, v. 66, p. 671-684.

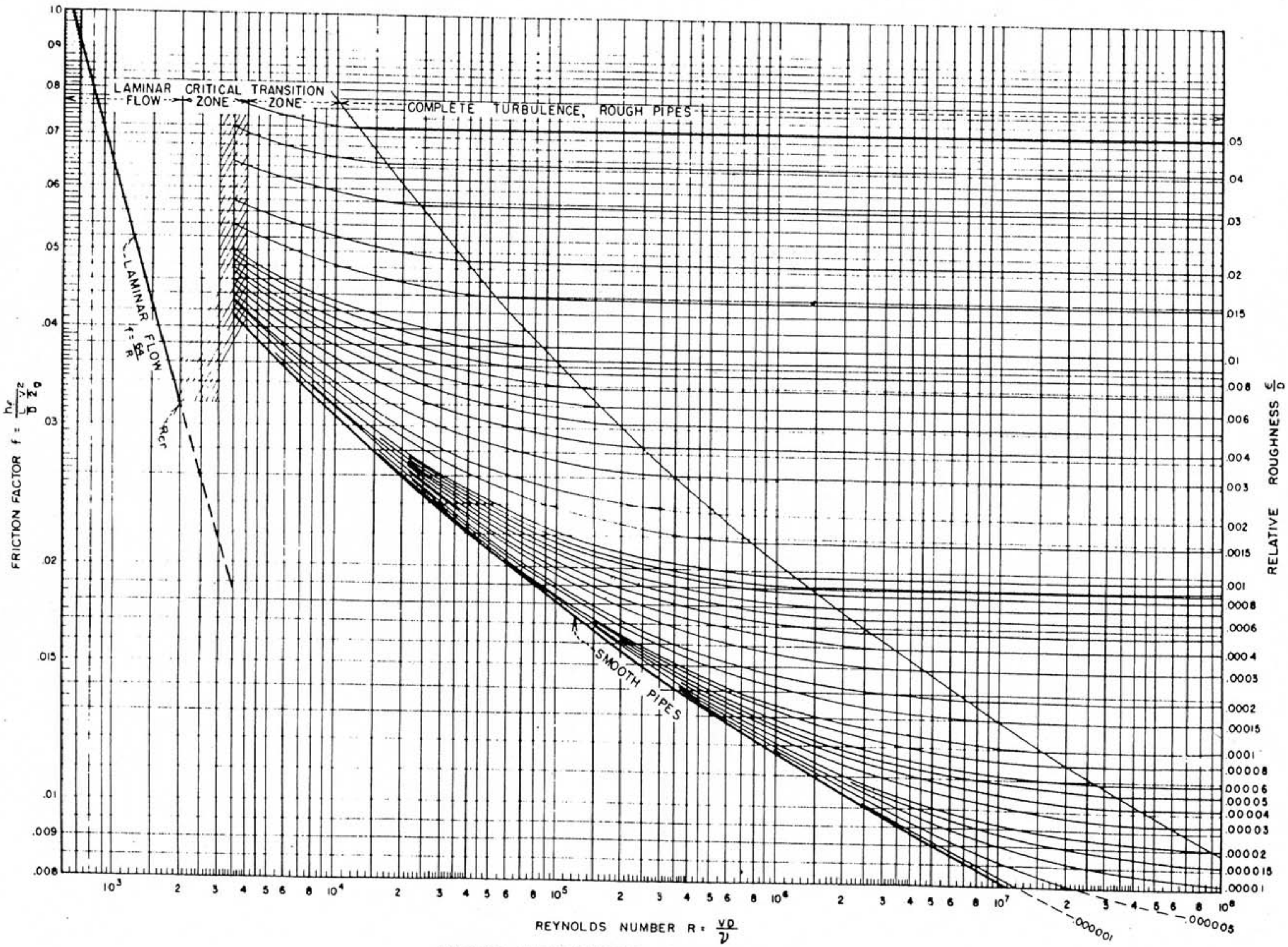
Rouse, H., 1946, Elementary Mechanics of Fluids, John Wiley and Sons, 376 p.

Rouse, H., 1958

Nikuradse, 1933

Brownlie, 1983

Barnes, H. H., 1967, Roughness characteristics of natural channels, Geological Survey Water Supply Paper 1869, 213 p.



MOODY DIAGRAM FOR FRICTION IN PIPES

NIKURADSE SAND GRAIN ROUGHNESS FOR CIRCULAR PIPES