

Definitions:

- **Polynomial:** A polynomial in x is any expression which can be written as:

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are integers and $a_n \neq 0$.

- **Degree:** The degree of a polynomial is the highest exponent of the polynomial.
- **Monomial:** is a polynomial with one term.
- **Binomial:** is a polynomial with two terms.
- **Trinomial:** is a polynomial with three terms.

Important Properties:

- **Dividing a Polynomial by a Monomial:** When dividing a polynomial by a monomial, divide each term of the polynomial by the monomial:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

- When dividing a polynomial by a monomial you will need to recall the Quotient Rule for Exponents: For any nonzero number a and any integers m and n ,

$$\frac{a^m}{a^n} = a^{m-n}.$$

When we divide like bases, we subtract the exponents.

- **Dividing Polynomials:** use long division.
- When a certain power of x is missing it is a good idea to put a zero as a place holder for this term.

Common Mistakes to Avoid:

- Do NOT stop the long division process until the remainder is a polynomial with one degree less than the divisor.

PROBLEMS

Perform the indicated operations and simplify.

1. $\frac{6x^2 + 9x - 12}{3x}$

$$\begin{aligned}\frac{6x^2 + 9x - 12}{3x} &= \frac{6x^2}{3x} + \frac{9x}{3x} - \frac{12}{3x} \\ &= \boxed{2x + 3 - \frac{4}{x}}\end{aligned}$$

2. $\frac{20x^4 - 12x^3 + 4x^2 - 6x + 8}{2x}$

$$\begin{aligned}\frac{20x^4 - 12x^3 + 4x^2 - 6x + 8}{2x} &= \frac{20x^4}{2x} - \frac{12x^3}{2x} + \frac{4x^2}{2x} - \frac{6x}{2x} + \frac{8}{2x} \\ &= \boxed{10x^3 - 6x^2 + 2x - 3 + \frac{4}{x}}\end{aligned}$$

3. $\frac{8x^4 - 12x^3 - 2x^2 + 2x - 7}{2x^2}$

$$\begin{aligned}\frac{8x^4 - 12x^3 - 2x^2 + 2x - 7}{2x^2} &= \frac{8x^4}{2x^2} - \frac{12x^3}{2x^2} - \frac{2x^2}{2x^2} + \frac{2x}{2x^2} - \frac{7}{2x^2} \\ &= \boxed{4x^2 - 6x - 1 + \frac{1}{x} - \frac{7}{2x^2}}\end{aligned}$$

4. $\frac{32x^4y^5 - 8x^3y^4 + 7x^2y^3 - 24xy^2 - 12xy^7}{-4x^2y^3}$

$$\begin{aligned}\frac{32x^4y^5 - 8x^3y^4 + 7x^2y^3 - 24xy^2 - 12xy^7}{-4x^2y^3} &= \frac{32x^4y^5}{-4x^2y^3} - \frac{8x^3y^4}{-4x^2y^3} + \frac{7x^2y^3}{-4x^2y^3} - \frac{24xy^2}{-4x^2y^3} - \frac{12xy^7}{-4x^2y^3} \\ &= \boxed{-8x^2y^2 + 2xy - \frac{7}{4} + \frac{6}{xy} + \frac{3y^4}{x}}\end{aligned}$$

5. $\frac{x^2 - 5x + 6}{x - 3}$

$$\begin{array}{r} x - 2 \\ x - 3) \overline{x^2 - 5x + 6} \\ \underline{x^2 - 3x} \\ \underline{-2x + 6} \\ \underline{-2x + 6} \\ 0 \end{array}$$

quotient = $x - 2$

6. $\frac{x^3 - x^2 - 2x + 6}{x - 2}$

$$\begin{array}{r} x^2 + x \\ x - 2) \overline{x^3 - x^2 - 2x + 6} \\ \underline{x^3 - 2x^2} \\ \underline{x^2 - 2x} \\ \underline{x^2 - 2x} \\ 6 \end{array}$$

quotient = $x^2 + x$, remainder = 6

7. $\frac{x^3 + 9x^2 - 5}{x^2 - 1}$

$$\begin{array}{r} x + 9 \\ x^2 + 0x - 1) \overline{x^3 + 9x^2 + 0x - 5} \\ \underline{x^3 + 0x^2 - x} \\ \underline{9x^2 + x - 5} \\ \underline{9x^2 + 0x - 9} \\ x + 4 \end{array}$$

quotient = $x + 9$, remainder = $x + 4$

8. $\frac{3x^4 - 5x^3 - 20x - 5}{x^2 + x + 3}$

$$\begin{array}{r} 3x^2 - 8x - 1 \\ x^2 + x + 3) \overline{3x^4 - 5x^3 + 0x^2 - 20x - 5} \\ \underline{3x^4 + 3x^3 + 9x^2} \\ \underline{-8x^3 - 9x^2 - 20x} \\ \underline{-8x^3 - 8x^2 - 24x} \\ \underline{-x^2 + 4x - 5} \\ \underline{-x^2 - x - 3} \\ 5x - 2 \end{array}$$

quotient = $3x^2 - 8x - 1$, remainder = $5x - 2$