**Definitions**:

- **Function**: A function $f$ is a rule that assigns to each element $x$ in the set $A$ exactly one element, called $f(x)$, in the set $B$. The set $A$ is called the **domain** and the set $B$ is called the **range**.
- **Domain**: The domain of a function is the set of all real numbers for which the expression is defined as a real number. In other words, it is all the real numbers for which the expression "makes sense."

**Important Properties**:

- Remember that you cannot have a zero in the denominator.
- Remember that you cannot have a negative number under an even root.
- Remember that we can evaluate an odd root of a negative number.

**Common Mistakes to Avoid**:

- Do not exclude from the domain the $x$ values which make the quantity under an odd root negative.

## PROBLEMS

Find the domain of each function.

1. $h(x) = \dfrac{x^3}{x^2 + 2x - 3}$

Since we cannot have a zero in the denominator, we will first find out which numbers make the denominator zero. To do this we will solve

$$x^2 + 2x - 3 = 0.$$

After solving this, we will exclude its solutions from the domain.

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x + 3 = 0 \qquad x = -3$$

$$x - 1 = 0 \qquad x = 1$$

**Domain: $x \neq -3, \quad x \neq 1$**

OR

**Domain: All real numbers except $-3$ and $1$**

2. $f(x) = \sqrt{6 - 4x}$

Remember that we cannot have a negative number under a square root. Therefore, $6 - 4x$ must be either positive or zero. Hence,

$$\begin{aligned} 6 - 4x &\geq 0 \\ -4x &\geq -6 \\ x &\leq \frac{-6}{-4} \\ x &\leq \frac{3}{2} \end{aligned}$$

$$\boxed{\text{Domain: } x \leq \frac{3}{2}}$$

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3. $f(x) = \dfrac{x - 1}{3x^2 + 2x - 2}$

Since we cannot have a zero in the denominator, we will find what values of $x$ make the denominator zero. In other words, we will solve

$$3x^2 + 2x - 2 = 0.$$

Once we find the solution, these numbers will then be excluded from the domain.

Since $3x^2 + 2x - 2$ does not factor, we will use the quadratic formula to solve. Here, $a = 3,\ b = 2$ and $c = -2$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4(3)(-2)}}{2(3)} \\ &= \frac{-2 \pm \sqrt{4 + 24}}{6} \\ &= \frac{-2 \pm \sqrt{28}}{6} \\ &= \frac{-2 \pm 2\sqrt{7}}{6} \\ &= \frac{2(-1 \pm \sqrt{7})}{6} \\ &= \frac{-1 \pm \sqrt{7}}{3} \end{aligned}$$

$$\boxed{\text{Domain: } x \neq \frac{-1 \pm \sqrt{7}}{3}}$$

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4. $g(x) = \dfrac{\sqrt{x}}{4x - 7}$

Since we cannot have zero in the denominator, we will need to find out what value of $x$ makes $4x - 7$ zero and exclude this from the domain.

$$\begin{aligned} 4x - 7 &= 0 \\ 4x &= 7 \\ x &= \frac{7}{4} \end{aligned}$$

In addition, we are also unable to take the square root of a negative number. Therefore, $x$ must be positive or zero. In other words, $x \geq 0$.

As a result of excluding $x = \frac{7}{4}$ and insisting that $x \geq 0$, we get the following answer.

$$\boxed{\text{Domain: } x \geq 0, \quad x \neq \frac{7}{4}}$$

5. $g(x) = \dfrac{\sqrt[3]{x+1}}{x^2-4}$

The first thing to note is that since we can evaluate the cube root of a positive, negative, or zero number, we do not need to make any restrictions from the numerator.

However, we once again cannot have zero in the denominator. Therefore, we must find the values of $x$ that make the denominator zero by solving

$$x^2 - 4 = 0.$$

Once we have solved this, we will eliminate its solutions from the domain.

$$\begin{aligned} x^2 - 4 &= 0 \\ x^2 &= 4 \\ \sqrt{x^2} &= \sqrt{4} \\ x &= \pm 2 \end{aligned}$$

$$\boxed{\text{Domain: } x \neq 2, \quad x \neq -2}$$

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6. $h(x) = \dfrac{2x-3}{\sqrt[4]{x-7}}$

For this problem, not only can we not have a negative under the 4-th root, but since the radical occurs in the denominator, we also cannot have a zero under it. Therefore, $x - 7 > 0$. Solving this, we get

$$\begin{aligned} x - 7 &> 0 \\ x &> 7 \end{aligned}$$

$$\boxed{\text{Domain: } x > 7}$$

7. $f(x) = \dfrac{7x+8}{6x^2 - 19x + 10}$

We must determine where the denominator is zero. To do this we will solve

$$6x^2 - 19x + 10 = 0.$$

Therefore,

$$\begin{aligned} 6x^2 - 19x + 10 &= 0 \\ (3x-2)(2x-5) &= 0 \end{aligned}$$

$$\begin{aligned} 3x - 2 &= 0 \\ 3x &= 2 \\ x &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} 2x - 5 &= 0 \\ 2x &= 5 \\ x &= \frac{5}{2} \end{aligned}$$

Excluding these values from the domain, we get

$$\boxed{\text{Domain: } x \neq \frac{2}{3}, \quad x \neq \frac{5}{2}}$$

8. $g(x) = \dfrac{3x^2 + 5x - 2}{\sqrt[3]{7 - 3x}}$

Remember that we are able to evaluate the cube root of a negative number. However, since the cube root is in the denominator, we are not allowed to let it be zero. As a result, we will find where the denominator is zero, and exclude this value from the domain.

$$7 - 3x = 0$$
$$-3x = -7$$
$$x = \frac{-7}{-3}$$
$$x = \frac{7}{3}$$

$$\boxed{\text{Domain: } x \neq \frac{7}{3}}$$

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9. $h(x) = \dfrac{\sqrt{2 + 7x}}{x^2 - 8x + 7}$

First, let us deal with the numerator. Because we have a square root, we are unable to take the square root of a negative number. Therefore, we will solve for where $2+7x \geq 0$.

$$2 + 7x \geq 0$$
$$7x \geq -2$$
$$x \geq \frac{-2}{7}$$

Next, we are not allowed to have a zero in the denominator. Therefore, we will solve for where $x^2 - 8x + 7 = 0$ and then eliminate these values from the domain.

$$x^2 - 8x + 7 = 0$$
$$(x - 7)(x - 1) = 0$$

$$x - 7 = 0$$
$$x = 7$$

$$x - 1 = 0$$
$$x = 1$$

Therefore, putting these together, we get

$$\boxed{\text{Domain: } x \geq \frac{-2}{7}, \quad x \neq 7, \quad x \neq 1}$$

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10. $g(x) = \dfrac{\sqrt[4]{3x + 2}}{9x - 5}$

Since we are not able to take the 4th root of a negative number, we need to solve for where $3x + 2 \geq 0$.

$$3x + 2 \geq 0$$
$$3x \geq -2$$
$$x \geq \frac{-2}{3}$$

Next, the denominator cannot be zero. Hence, we will solve for where $9x - 5 = 0$ and eliminate this value from the domain.

$$9x - 5 = 0$$
$$9x = 5$$
$$x = \frac{5}{9}$$

Putting these together, we get

$$\boxed{\text{Domain: } x \geq \frac{-2}{3}, \quad x \neq \frac{5}{9}}$$