

Definitions:

- **Polynomial:** A polynomial in x is any expression which can be written as:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n, a_{n-1}, \dots, a_1, a_0$ are integers and $a_n \neq 0$.

- **Degree:** The degree of a polynomial is the highest exponent of the polynomial.
- **Monomial:** is a polynomial with one term.
- **Binomial:** is a polynomial with two terms.
- **Trinomial:** is a polynomial with three terms.
- **like terms:** means the same variable to the same power. For example, $2x^2$ and $3x^2$ are like terms because they have the same variable raised to the same power. However, $2x^2$ and $3x^3$ are not like terms because the powers are different.

Important Properties:

- To add or subtract polynomials we combine like terms.
- To multiply polynomials we multiply each term in the first polynomial by each term in the second polynomial. In order to do this, we need to recall the product rule for exponents: For any integers m and n ,

$$a^m \cdot a^n = a^{m+n}.$$

- **FOIL:** First Outer Innner Last. This is one method for multiplying two binomials.
- **Distributive Property:** Recall that

$$a(b + c) = ab + ac \quad \text{and} \quad a(b - c) = ab - ac$$

- **Special Formulas:** Note that you do not need to memorize these formulas. They arise by using FOIL.

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x - y)(x + y) = x^2 - y^2$$

Common Mistakes to Avoid:

- NOTE: $(x+y)^2 \neq x^2 + y^2$. In fact, $(x+y)^n \neq x^n + y^n$ for all integers $n \geq 2$. To avoid this mistake use the exponent to rewrite the expression before multiplying. For example,

$$(x+y)^3 = (x+y)(x+y)(x+y).$$

- Be careful when combining like terms. Remember that the variable part remains unchanged. For example,

$$2x^2 + 5x^2 = 7x^2 \quad \text{and not} \quad 2x^2 + 5x^2 \neq 7x^4.$$

PROBLEMS

Perform the indicated operations and simplify:

1. $(3x^3 - 7x^2 + 5x - 3) + (8x^3 + 2x^2 - 6x + 1)$

$$\begin{array}{r} (3x^3 - 7x^2 + 5x - 3) + (8x^3 + 2x^2 - 6x + 1) \\ 3x^3 + 8x^3 - 7x^2 + 2x^2 + 5x - 6x - 3 + 1 \\ \boxed{11x^3 - 5x^2 - x - 2} \end{array}$$

2. $(5x^4 - 3x^3 + 2x - 7) - (2x^4 + 3x^3 - 4x^2 + 6x + 9)$

$$\begin{array}{r} (5x^4 - 3x^3 + 2x - 7) - (2x^4 + 3x^3 - 4x^2 + 6x + 9) \\ 5x^4 - 3x^3 + 2x - 7 - 2x^4 - 3x^3 + 4x^2 - 6x - 9 \\ 5x^4 - 2x^4 - 3x^3 - 3x^3 + 4x^2 + 2x - 6x - 7 - 9 \\ \boxed{3x^4 - 6x^3 + 4x^2 - 4x - 16} \end{array}$$

3. $(3x^2 + x - 3) - (2x^2 - 5x + 5) - (-7x^2 - 5x + 6)$

$$\begin{array}{r} (3x^2 + x - 3) - (2x^2 - 5x + 5) - (-7x^2 - 5x + 6) \\ 3x^2 + x - 3 - 2x^2 + 5x - 5 + 7x^2 + 5x - 6 \\ 3x^2 - 2x^2 + 7x^2 + x + 5x + 5x - 3 - 5 - 6 \\ \boxed{8x^2 + 11x - 14} \end{array}$$

4. $(3x^2 + 4x - 7) - (2x^3 - 5x + 2) - (3x^2 - 4x)$

$$\begin{array}{r} (3x^2 + 4x - 7) - (2x^3 - 5x + 2) - (3x^2 - 4x) \\ 3x^2 + 4x - 7 - 2x^3 + 5x - 2 - 3x^2 + 4x \\ -2x^3 + 3x^2 - 3x^2 + 4x + 5x + 4x - 7 - 2 \\ \boxed{-2x^3 + 13x - 9} \end{array}$$

5. $(9x^2y - 3x^2 + 2y) - (4x^2y - 4x^2 - 3y)$

$$\begin{array}{r} (9x^2y - 3x^2 + 2y) - (4x^2y - 4x^2 - 3y) \\ 9x^2y - 3x^2 + 2y - 4x^2y + 4x^2 + 3y \\ 9x^2y - 4x^2y - 3x^2 + 4x^2 + 2y + 3y \\ \boxed{5x^2y + x^2 + 5y} \end{array}$$

6. $(6x^3y^2 - 3x^2y^3 + 7x^2y^2) + (5x^2y^3 - 8x^2y^2 + 12x^3y^2)$

$$\begin{array}{r} 6x^3y^2 - 3x^2y^3 + 7x^2y^2 + 5x^2y^3 - 8x^2y^2 + 12x^3y^2 \\ 6x^3y^2 + 12x^3y^2 - 3x^2y^3 + 5x^2y^3 + 7x^2y^2 - 8x^2y^2 \\ \boxed{18x^3y^2 + 2x^2y^3 - x^2y^2} \end{array}$$

7. $2x(3x^2 - 5x + 4)$

$$\begin{array}{r} 2x(3x^2 - 5x + 4) \\ 6x^{1+2} - 10x^{1+1} + 8x \\ \boxed{6x^3 - 10x^2 + 8x} \end{array}$$

8. $5x^2(7x^3 - 5x + 3x - 2)$

$$\begin{array}{r} 5x^2(7x^3 - 5x + 3x - 2) \\ 35x^{2+3} - 25x^{2+2} + 15x^{2+1} - 10x^2 \\ \boxed{35x^5 - 25x^4 + 15x^3 - 10x^2} \end{array}$$

9. $3xy^2(4xy^3 - 5x^2y + 3xy)$

$$\begin{array}{r} 3xy^2(4xy^3 - 5x^2y + 3xy) \\ 12x^{1+1}y^{2+3} - 15x^{1+2}y^{2+1} + 9x^{1+1}y^{2+1} \\ \boxed{12x^2y^5 - 15x^3y^3 + 9x^2y^3} \end{array}$$

10. $(2x - 5)(3x + 2)$

$$\begin{array}{r} (2x - 5)(3x + 2) \\ 6x^2 + 4x - 15x - 10 \\ \boxed{6x^2 - 11x - 10} \end{array}$$

11. $(2x^2 - 5)(3x^2 + 1)$

$$\begin{array}{r} (2x^2 - 5)(3x^2 + 1) \\ 5x^4 + 2x^2 - 15x^2 - 5 \\ \boxed{5x^4 - 13x^2 - 5} \end{array}$$

12. $(3x^2 - 4)(2x + 7)$

$$\begin{array}{r} (3x^2 - 4)(2x + 7) \\ \boxed{6x^3 + 21x^2 - 8x - 28} \end{array}$$

13. $(2x - 1)(3x^2 + 4x + 2)$

$$\begin{array}{r} (2x - 1)(3x^2 + 4x + 2) \\ 6x^3 + 8x^2 + 4x - 3x^2 - 4x - 2 \\ 6x^3 + 8x^2 - 3x^2 + 4x - 4x - 2 \\ \boxed{6x^3 + 5x^2 - 2} \end{array}$$

14. $(3xy - 2)(4xy + x^2y + 4)$

$$\begin{array}{r} (3xy - 2)(4xy + x^2y + 4) \\ 12x^2y^2 + 3x^3y + 12xy - 8xy - 2x^2y - 8 \\ \boxed{12x^2y^2 + 3x^3y + 4xy - 2x^2y - 8} \end{array}$$

15. $(2x - 3)^2$

$$\begin{array}{r} (2x - 3)^2 \\ (2x - 3)(2x - 3) \\ 4x^2 - 6x - 6x + 9 \\ \boxed{4x^2 - 12x + 9} \end{array}$$

16. $(x - 2)^3$

$$\begin{array}{r} (x - 2)^3 \\ (x - 2)(x - 2)(x - 2) \\ (x^2 - 2x - 2x + 4)(x - 2) \\ (x^2 - 4x + 4)(x - 2) \\ x^3 - 2x^2 - 4x^2 + 8x + 4x - 8 \\ \boxed{x^3 - 6x^2 + 12x - 8} \end{array}$$

17. $(2x - 3)^4$

$$\begin{aligned}
 & (2x - 3)^4 \\
 & (2x - 4)(2x - 4)(2x - 4)(2x - 4) \\
 & (4x^2 - 8x - 8x + 16)(2x - 4)(2x - 4) \\
 & (4x^2 - 16x + 16)(2x - 4)(2x - 4) \\
 & (8x^3 - 16x^2 - 32x^2 + 64x + 32x - 64)(2x - 4) \\
 & (8x^3 - 48x^2 + 96x - 64)(2x - 4) \\
 & 16x^4 - 32x^3 - 96x^3 + 192x^2 + 192x^2 - 384x - 128x + 256 \\
 & \boxed{16x^4 - 128x^3 + 384x^2 - 512x + 256}
 \end{aligned}$$