Definition:

- **Logarithmic function**: Let a be a positive number with $a \neq 1$. The logarithmic function with base a, denoted $\log_a x$, is defined by

$$\boxed{y = \log_a x \qquad \text{if and only if} \qquad x = a^y.}$$

Laws of Logarithms: Let a be a positive number with $a \neq 1$. Let $A > 0$, $B > 0$, and n be any real number.

1. $\boxed{\log_a AB = \log_a A + \log_a B.}$ (The logarithm of a product is the sum of the logarithms.)
2. $\boxed{\log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B.}$ (The logarithm of a quotient is the difference of the logarithms.)
3. $\boxed{\log_a A^n = n \log_a A.}$ (The logarithm of a quantity raised to a power is the same as the power times the logarithm of the quantity.)

Important Properties:

- **Change of base formula**:

$$\boxed{\log_b x = \frac{\log_a x}{\log_a b}.}$$

- The change of base formula allows you to use your calculator to evaluate logarithms. In order to use the calculator, a must be either 10 or e.

Common Mistakes to Avoid:

- $\log_a(A \pm B) \neq \log_a A \pm \log_a B$. In other words, there is no law of logarithms corresponding to the logarithm of a sum or difference.
- $(\log_a A)^n \neq n \log_a A$. When the entire logarithm is raised to the n-th power, you cannot use the 3rd law of logarithms to bring down the exponent.
- $\log_a A - \log_a B \neq \frac{\log_a A}{\log_a B}$. The difference of the logarithms is not the same as the quotient of the logarithms.
- $\log_a AB \neq (\log_a A)(\log_a B)$. The logarithm of a product is equal to the sum of the logarithms NOT the product of the logarithms.

PROBLEMS

1. **Use the laws of logarithms to rewrite the expression in a form with no logarithm of a product, quotient, or power.**

   (a) $\log_2 x^3(x-4)^7$

$$\begin{aligned}\log_2 x^3(x-4)^7 &= \log_2 x^3 + \log_2(x-4)^7 \\ &= 3\log_2 x + 7\log_2(x-4)\end{aligned}$$

$$\boxed{3\log_2 x + 7\log_2(x-4)}$$

   (b) $\log\left(\dfrac{x^2(x+1)^6}{(x-3)^5}\right)$

$$\begin{aligned}\log\left(\frac{x^2(x+1)^6}{(x-3)^5}\right) &= \log x^2(x+1)^6 - \log(x-3)^5 \\ &= \log x^2 + \log(x+1)^6 - \log(x-3)^5 \\ &= 2\log x + 6\log(x+1) - 5\log(x-3)\end{aligned}$$

$$\boxed{2\log x + 6\log(x+1) - 5\log(x-3)}$$

   (c) $\log_3\left(\dfrac{x^3}{\sqrt{x+1}(x-9)^7}\right)$

$$\begin{aligned}\log_3\left(\frac{x^3}{\sqrt{x+1}(x-9)^7}\right) &= \log_3 x^3 - \log_3 \sqrt{x+1}(x-9)^7 \\ &= \log_3 x^3 - \log_3\sqrt{x+1} - \log_3(x-9)^7 \\ &= 3\log_3 x - \frac{1}{2}\log_3(x+1) - 7\log_3(x-9)\end{aligned}$$

$$\boxed{3\log_3 x - \frac{1}{2}\log_3(x+1) - 7\log_3(x-9)}$$

(d) $\log_5 \left(\frac{x(x-4)^2}{(x+1)^3} \right)^2$

$$\begin{aligned} \log_5 \left(\frac{x(x-4)^2}{(x+1)^3} \right)^2 &= \log_5 \frac{x^2(x-4)^4}{(x+1)^6} \\ &= \log_5 x^2(x-4)^4 - \log_5 (x+1)^6 \\ &= \log_5 x^2 + \log_5 (x-4)^4 - \log_5 (x+1)^6 \\ &= 2\log_5 x + 4\log_5 (x-4) - 6\log_5 (x+1) \end{aligned}$$

$$\boxed{2\log_5 x + 4\log_5 (x-4) - 6\log_5 (x+1)}$$

(e) $\ln \left(\frac{x^4 \sqrt[3]{z}}{\sqrt{y^2+3}} \right)$

$$\begin{aligned} \ln \left(\frac{x^4 \sqrt[3]{z}}{\sqrt{y^2+3}} \right) &= \ln x^4 \sqrt[3]{z} - \ln \sqrt{y^2+3} \\ &= \ln x^4 + \ln \sqrt[3]{z} - \ln \sqrt{y^2+3} \\ &= 4\ln x + \frac{1}{3}\ln z - \frac{1}{2}\ln(y^2+3) \end{aligned}$$

$$\boxed{4\ln x + \frac{1}{3}\ln z - \frac{1}{2}\ln(y^2+3)}$$

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2. **Rewrite the expression as a single logarithm.**

(a) $5\log z - 3\log x + 7\log y$

$$\begin{aligned} 5\log z - 3\log x + 7\log y &= \log z^5 - \log x^3 + \log y^7 \\ &= \log \frac{z^5}{x^3} + \log y^7 \\ &= \log \frac{z^5 y^7}{x^3} \end{aligned}$$

$$\boxed{\log \frac{z^5 y^7}{x^3}}$$

(b) $3\ln(x-2) - 5\left[\ln x - 2\ln(x+1)\right]$

$$\begin{aligned} 3\ln(x-2) - 5\left[\ln x - 2\ln(x+1)\right] &= 3\ln(x-2) - 5\ln x + 10\ln(x+1) \\ &= \ln(x-2)^3 - \ln x^5 + \ln(x+1)^{10} \\ &= \ln\frac{(x-2)^3}{x^5} + \ln(x+1)^{10} \\ &= \ln\frac{(x-2)^3(x+1)^{10}}{x^5} \end{aligned}$$

$$\boxed{\ln\frac{(x-2)^3(x+1)^{10}}{x^5}}$$

(c) $\frac{1}{4}\log(3x-2) + \frac{1}{2}\left[\log(x-2) - \log(x+7)\right]$

$$\begin{aligned} \frac{1}{4}\log(3x-2) + \frac{1}{2}\left[\log(x-2) - \log(x+7)\right] &= \frac{1}{4}\log(3x-2) + \frac{1}{2}\log\frac{x-2}{x+7} \\ &= \log\sqrt[4]{3x-2} + \log\sqrt{\frac{x-2}{x+7}} \\ &= \log\sqrt[4]{3x-2}\sqrt{\frac{x-2}{x+7}} \end{aligned}$$

$$\boxed{\log\sqrt[4]{3x-2}\sqrt{\frac{x-2}{x+7}}}$$

(d) $4\left[\ln x - \ln(x+5)\right] - 2\ln(x-5)$

$$\begin{aligned} 4\left[\ln x - \ln(x+5)\right] - 2\ln(x-5) &= 4\ln x - 4\ln(x+5) - 2\ln(x-5) \\ &= \ln x^4 - \ln(x+5)^4 - \ln(x-5)^2 \\ &= \ln\frac{x^4}{(x+5)^4} - \ln(x-5)^2 \\ &= \ln\frac{x^4}{(x+5)^4(x-5)^2} \end{aligned}$$

$$\boxed{\ln\frac{x^4}{(x+5)^4(x-5)^2}}$$

3. **Use the change of base formula and a calculator to evaluate the logarithm correct to four decimal places.**

   (a) $\log_7 3$

   We will switch to base 10. Therefore,

$$\log_7 3 = \frac{\log 3}{\log 7} = .5645750341.$$

$$\boxed{\log_7 3 = .5646}$$

   (b) $\log_{12} 8$

   This time we will switch to base e. Thus,

$$\log_{12} 8 = \frac{\ln 8}{\ln 12} = .836828837.$$

$$\boxed{\log_{12} 8 = .8368}$$