Definition:

- **Quadratic-Type Expression:** is an expression of the form $au^2 + bu + c$ where u is an algebraic expression. For example, the following are all quadratic-type expressions:

$$x^4 + 5x^2 + 6, \qquad (x-3)^2 - 2(x-3) - 3, \qquad x^8 - 7x^4 - 18.$$

Important Properties:

- If you recognize how the expression factors in its original form, then do so. If not, then use a u substitution to create a quadratic that is easier to factor. For example, for $x^4 + 5x^2 + 6$, if we let $u = x^2$ then we have

$$x^4 + 5x^2 + 6 = u^2 + 5u + 6 = (u+3)(u+2) = (x^2+2)(x^2+3).$$

- Only use the substitution if it helps you factor the expression.

Common Mistakes to Avoid:

- If you use the u substitution, you must substitute back to the original variable. So, if the original problem is in terms of x, make sure that your answer is also in terms of x.
- Be on the lookout for the difference of squares, the difference of cubes, and the sum of cubes. Remember that these can be factored further.

PROBLEMS

Factor Completely.

1. $x^4 - 5x^2 - 6$

$$x^4 - 5x^2 - 6$$
$$\boxed{(x^2-6)(x^2+1)}$$

NOTE: You could also use a u substitution on this problem. Letting $u = x^2$, we get

$$x^4 - 5x^2 - 6$$
$$u^2 - 5u - 6$$
$$(u-6)(u+1)$$
$$\boxed{(x^2-6)(x^2+1)}$$

2. $x^4 + 5x^2 - 36$

$$x^4 + 5x^2 - 36$$
$$(x^2-4)(x^2+9)$$
$$\boxed{(x-2)(x+2)(x^2+9)}$$

OR

Letting $u = x^2$, we get

$$x^4 + 5x^2 - 36$$
$$u^2 + 5u - 36$$
$$(u-4)(u+9)$$
$$(x^2-4)(x^2+9)$$
$$\boxed{(x-2)(x+2)(x^2+9)}$$

3. $x^4 + 15x^2 - 16$

$$x^4 + 15x^2 - 16$$
$$(x^2 + 16)(x^2 - 1)$$
$$\boxed{(x^2 + 16)(x - 1)(x + 1)}$$

OR

Letting $u = x^2$, we get

$$x^4 + 15x^2 - 16$$
$$u^2 + 15u - 16$$
$$(u + 16)(u - 1)$$
$$(x^2 + 16)(x^2 - 1)$$
$$\boxed{(x^2 + 16)(x - 1)(x + 1)}$$

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4. $x^8 - 7x^4 - 18$

$$x^8 - 7x^4 - 18$$
$$(x^4 - 9)(x^4 + 2)$$
$$\boxed{(x^2 - 3)(x^2 + 3)(x^4 + 2)}$$

OR

Letting $u = x^4$, we get

$$x^8 - 7x^4 - 18$$
$$u^2 - 7u - 18$$
$$(u - 9)(u + 2)$$
$$(x^4 - 9)(x^4 + 2)$$
$$\boxed{(x^2 - 3)(x^2 + 3)(x^4 + 2)}$$

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5. $x^6 + 4x^3 - 45$

$$x^6 + 4x^3 - 45$$
$$\boxed{(x^3 + 9)(x^3 - 5)}$$

OR

Letting $u = x^3$, we get

$$x^6 + 4x^3 - 45$$
$$u^2 + 4u - 45$$
$$(u + 9)(u - 5)$$
$$\boxed{(x^3 + 9)(x^3 - 5)}$$

6. $6x^6 - 19x^3 - 7$

$$6x^6 - 19x^3 - 7$$
$$\boxed{(3x^3 + 1)(2x^3 - 7)}$$

OR

Letting $u = x^3$, we get

$$6x^6 - 19x^3 - 7$$
$$6u^2 - 19u - 7$$
$$(3u + 1)(2u - 7)$$
$$\boxed{(3x^3 + 1)(2x^3 - 7)}$$

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7. $5x^{10} - 19x^5 + 12$

$$5x^{10} - 19x^5 + 12$$
$$\boxed{(5x^5 - 4)(x^5 - 3)}$$

OR

Letting $u = x^5$, we get

$$5x^{10} - 19x^5 + 12$$
$$5u^2 - 19u + 12$$
$$(5u - 4)(u - 3)$$
$$\boxed{(5x^5 - 4)(x^5 - 3)}$$

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8. $(x - 3)^2 - 2(x - 3) - 3$

$$(x - 3)^2 - 2(x - 3) - 3$$
$$[(x - 3) - 3][(x - 3) + 1]$$
$$(x - 3 - 3)(x - 3 + 1)$$
$$\boxed{(x - 6)(x - 2)}$$

OR

Letting $u = (x - 3)$, we get

$$(x - 3)^2 - 2(x - 3) - 3$$
$$u^2 - 2u - 3$$
$$(u - 3)(u + 1)$$
$$[(x - 3) - 3][(x - 3) + 1]$$
$$(x - 3 - 3)(x - 3 + 1)$$
$$\boxed{(x - 6)(x - 2)}$$