

Definitions:

- **Whole Numbers:** $\{0, 1, 2, 3, \dots\}$
- **Integers:** $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- **Rational Numbers:** Any number that can be written as a fraction or whose decimal expansion either terminates or repeats.
- **Irrational Numbers:** Any number that cannot be written as a fraction or whose decimal expansion does not terminate nor repeat.
- **Real Numbers:** The collection of all rational and irrational numbers.
- **Absolute Value:** The absolute value of a real number n , denoted $|n|$, is the distance between n and 0 on the number line.
- **Exponents:** An exponent is a number that tells how many times a factor is repeated in a product. For example,

$$2^4 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{4 \text{ times}} = 16.$$

Important Properties:

- **Order of Operations:** Parentheses, Exponents, Multiplication and Division, Addition and Subtraction; PEMDAS. This can be remembered with the mnemonic device *please excuse my dear aunt Sally*.
- Since $|n|$ is a distance, $|n|$ is always either positive or zero. So, for any real number, $|n| \geq 0$.

Common Mistakes to Avoid:

- In the order of operations, multiplication and division are performed from left to right as they occur. For example, in the problem $12 \div 3 \cdot 2$, we first divide 12 by 3 to get 4 and then multiply by 2.
- In the order of operations, addition and subtraction are performed from left to right as they occur. For example, in the problem $4 - 8 + 3$, we first subtract 8 from 4 to get -4 and then add 3.
- Do not distribute a number inside a quantity that is raised to a power. You must first raise the quantity to the given power before you can distribute.
- Never distribute a negative number inside an absolute value. To be safe, do not distribute any number inside an absolute value. Evaluate the absolute value first and then distribute.

PROBLEMS

Find the value of each expression.

1. $8 \div 4 + 3^2$

$$\begin{aligned} 8 \div 4 + 3^2 &= 8 \div 4 + 9 \\ &= 2 + 9 \\ &= 11 \end{aligned}$$

Answer: 11

2. $\frac{2(11 + 3)}{|-4|}$

$$\begin{aligned} \frac{2(11 + 3)}{|-4|} &= \frac{2(14)}{4} \\ &= \frac{28}{4} \\ &= 7 \end{aligned}$$

Answer: 7

3. $5 \cdot 14 - 8 \div 2$

$$\begin{aligned} 5 \cdot 14 - 8 \div 2 &= 70 - 4 \\ &= 66 \end{aligned}$$

Answer: 66

4. $3 \cdot 2^2 - 4 \cdot 5$

$$\begin{aligned} 3 \cdot 2^2 - 4 \cdot 5 &= 3 \cdot 4 - 4 \cdot 5 \\ &= 12 - 20 \\ &= -8 \end{aligned}$$

Answer: -8

5. $-3[2 + 3(5 - 2)]$

$$\begin{aligned} -3[2 + 3(5 - 2)] &= -3[2 + 3(3)] \\ &= -3[2 + 9] \\ &= -3(11) \\ &= -33 \end{aligned}$$

Answer: -33

6. $25\left(\frac{4}{5}\right) + 3^3 - 32 \div 2$

$$\begin{aligned} 25\left(\frac{4}{5}\right) + 3^3 - 32 \div 2 &= \frac{100}{5} + 27 - 16 \\ &= 20 + 27 - 16 \\ &= 47 - 16 \\ &= 31 \end{aligned}$$

Answer: 31

$$7. \frac{4 - 6(8 - 5)^2}{3^2 + 1}$$

$$\begin{aligned} \frac{4 - 6(8 - 5)^2}{3^2 + 1} &= \frac{4 - 6(3)^2}{3^2 + 1} \\ &= \frac{4 - 6(9)}{9 + 1} \\ &= \frac{4 - 54}{10} \\ &= \frac{-50}{10} \\ &= -5 \end{aligned}$$

$$\boxed{\text{Answer: } -5}$$

$$8. \frac{3(9 - 4) + 5(8 - 3)}{2^3 - (5 - 3)}$$

$$\begin{aligned} \frac{3(9 - 4) + 5(8 - 3)}{2^3 - (5 - 3)} &= \frac{3(5) + 5(5)}{2^3 - (2)} \\ &= \frac{15 + 25}{8 - 2} \\ &= \frac{40}{6} \\ &= \frac{20}{3} \end{aligned}$$

$$\boxed{\text{Answer: } \frac{20}{3}}$$

$$9. \frac{12^2 + 2^2 - 8}{10^2 - (-4)(-15)}$$

$$\begin{aligned} \frac{12^2 + 2^2 - 8}{10^2 - (-4)(-15)} &= \frac{144 + 4 - 8}{100 - (60)} \\ &= \frac{140}{40} \\ &= \frac{7}{2} \end{aligned}$$

$$\boxed{\text{Answer: } \frac{7}{2}}$$

$$10. \frac{6 \cdot 3 - 3 \cdot 4}{-2 \cdot 5 + 7(-3) - 10}$$

$$\begin{aligned} \frac{6 \cdot 3 - 3 \cdot 4}{-2 \cdot 5 + 7(-3) - 10} &= \frac{18 - 12}{-10 + (-21) - 10} \\ &= \frac{6}{-31 - 10} \\ &= \frac{6}{-41} \end{aligned}$$

$$\boxed{\text{Answer: } -\frac{6}{41}}$$