Topic 1: Tautologies

• Tautology: A tautology is a statement which is always true.

Example 1: Determine whether $p \lor \sim p$ is a tautology.

$$\begin{array}{c|c|c} p & \sim p & p \lor \sim p \\ \hline T & & & \\ F & & & \end{array}$$

Example 2: Construct a truth table for $\sim (p \lor r) \lor (p \lor q)$ and state whether it is a tautology.

p	q	r	$p \vee r$	$ \sim (p\vee r) $	$p \vee q$	$ \sim (p\vee r)\vee (p\vee q) $
Т	Т	Т				
Τ	Т	F				
Τ	F	T				
Τ	F	F				
F	Т	T				
F	Т	F				
F	F	T				
F	F	F				

Example 3: Construct a truth table for each of the following and state whether it is a tautology.

(a)
$$\sim p \lor (\sim q \land \sim p)$$

(b)
$$(p \to q) \lor (q \to p)$$