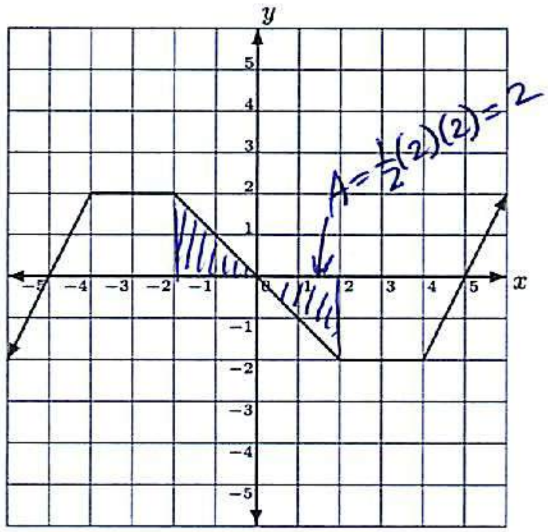
MATH 11012 FALL 2012 EXAM 3 (180 pts) NAME: Key

SHOW ALL WORK FOR FULL CREDIT — SIMPLIFY ALL ANSWERS — CIRCLE FINAL ANSWER
GIVE EXACT ANSWERS FOR ALL PROBLEMS — NO DECIMAL ANSWERS

1. For the function $y = f(x)$ graphed below, find the following:



(a) (0.5 pt) $\int_0^2 f(x)\,dx = \boxed{-2}$

(b) (0.5 pt) $\int_{-4}^{-2} f(x)\,dx = \boxed{4}$

(c) (0.5 pt) $\int_{-2}^{2} f(x)\,dx = \boxed{0}$

(d) (0.5 pt) $\int_{-2}^{1} f(x)\,dx = \boxed{\frac{3}{2}}$

(e) (0.5 pt) $\int_{-2}^{3} f(x)\,dx = \boxed{-2}$

Exam Score: $\frac{}{30} = \frac{}{180}$ Current Grade: $\frac{}{762} =$

2. (1 pt) Find: $\int \left(\frac{3}{4\sqrt{x}} - \frac{6}{\sqrt[3]{x}} \right) dx$

$= \int \left(\frac{3}{4} x^{-1/2} - 6x^{-1/3} \right) dx$

$= \frac{3}{4}(2) x^{1/2} - 6\left(\frac{3}{2}\right) x^{2/3} + C$

$= \boxed{\frac{3}{2} x^{1/2} - 9x^{2/3} + C}$

3. (1.5 pts) Find: $\int x^4 e^{x^5+3}\,dx$

let $u = x^5 + 3$

$du = 5x^4\,dx$

$\frac{1}{5} du = x^4\,dx$

$= \int \frac{1}{5} e^u\,du$

$= \frac{1}{5} e^u + C$

$= \boxed{\frac{1}{5} e^{x^5+3} + C}$

4. (2 pts) Find: $\int \frac{8x^3 - 6x^2 + 7x - 2}{x^2}\,dx$ $= \int \left(\frac{8x^3}{x^2} - \frac{6x^2}{x^2} + \frac{7x}{x^2} - \frac{2}{x^2} \right) dx$

$$= \int \left(8x - 6 + 7x^{-1} - 2x^{-2} \right) dx$$

$$= \boxed{4x^2 - 6x + 7\ln|x| + 2x^{-1} + C}$$

5. (1.5 pts) Find: $\int \frac{x^3 - 3x}{(x^4 - 6x^2 + 5)^6}\,dx$ $= \int (x^3 - 3x)(x^4 - 6x^2 + 5)^{-6}\,dx$

let $u = x^4 - 6x^2 + 5$

$du = (4x^3 - 12x)\,dx$

$du = 4(x^3 - 3x)\,dx$

$\frac{1}{4}du = (x^3 - 3x)\,dx$

$$= \int \frac{1}{4} u^{-6}\,du$$

$$= \frac{1}{4}\left(-\frac{1}{5}\right) u^{-5} + C$$

$$= \boxed{-\frac{1}{20}(x^4 - 6x^2 + 5)^{-5} + C}$$

6. (1.5 pts) Find: $\int \left(6x^{2/3} + 8x^{-3/2} - 4x^{-3/4}\right)\, dx$

$$= 6\left(\frac{3}{5}\right)x^{5/3} + 8(-2)x^{-1/2} - 4(4)x^{1/4} + C$$

$$= \boxed{\frac{18}{5}x^{5/3} - 16x^{-1/2} - 16x^{1/4} + C}$$

7. (2 pts) Find: $\int (18x^3 - 12x^2 + 6x - 5)\, dx$

$$= \frac{18}{4}x^4 - 4x^3 + 3x^2 - 5x + C$$

$$= \boxed{\frac{9}{2}x^4 - 4x^3 + 3x^2 - 5x + C}$$

8. (1 pt) Find: $\int \left(\frac{3}{4}x^5 - \frac{7}{8}x^{5/2}\right)\, dx = \frac{3}{4}\left(\frac{1}{6}\right)x^6 - \frac{7}{8}\left(\frac{2}{7}\right)x^{7/2} + C$

$$= \boxed{\frac{1}{8}x^6 - \frac{1}{4}x^{7/2} + C}$$

9. (1 pt) Find: $\int_3^7 \frac{3}{x^2}\,dx$ $= \int_3^7 3x^{-2}dx = -3x^{-1}\Big|_3^7$

$= \frac{-3}{x}\Big|_{x=3}^{x=7} = \frac{-3}{7} - \left(\frac{-3}{3}\right) = \frac{-3}{7} + 1 = \frac{-3}{7} + \frac{7}{7}$

$= \boxed{\frac{4}{7}}$

10. (1.5 pts) Find: $\int (4x+2)\sqrt{x^2+x-4}\,dx$ $= \int 2(2x+1)(x^2+x-4)^{1/2}dx$

let $u = x^2+x-4$
$du = (2x+1)\,dx$
$2du = 2(2x+1)\,dx$

$= \int 2u^{1/2}du$

$= 2\left(\frac{2}{3}\right)u^{3/2} + C$

$= \boxed{\frac{4}{3}(x^2+x-4)^{3/2} + C}$

11. (2 pts) Find: $\int_0^2 (x-3)(x^2-6x+5)^2\,dx = \int_{x=0}^{x=2} \frac{1}{2}u^2\,du$

let $u = x^2 - 6x + 5$

$du = (2x-6)\,dx$

$du = 2(x-3)\,dx$

$\frac{1}{2}du = (x-3)\,dx$

$= \frac{1}{2}\cdot\frac{1}{3}u^3 \Big|_{x=0}^{x=2}$

$= \frac{1}{6}(x^2-6x+5)^3 \Big|_{x=0}^{x=2}$

$= \frac{1}{6}(4-12+5)^3 - \frac{1}{6}(0^2-0+5)^3$

$= \frac{1}{6}(-3)^3 - \frac{1}{6}(5)^3$

$= -\frac{27}{6} - \frac{125}{6} = -\frac{152}{6} = \boxed{\frac{-76}{3}}$

12. (2 pts) Find: $\int_0^4 \frac{x}{\sqrt{x^2+9}}\,dx$ $= \int_0^4 x(x^2+9)^{-1/2}\,dx$

let $u = x^2+9$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \int_{x=0}^{x=4} \frac{1}{2} u^{-1/2}\,du$

$= \frac{1}{2}(2)u^{1/2} \Big|_{x=0}^{x=4} = \sqrt{x^2+9} \Big|_{x=0}^{x=4}$

$= \sqrt{16+9} - \sqrt{0+9}$

$= \sqrt{25} - \sqrt{9} = 5-3 = \boxed{2}$

13. (1.5 pts) Find: $\int_{-8}^{8} (2x + \sqrt[3]{x})\, dx$ $= \int_{-8}^{8} (2x + x^{1/3})\, dx$

$= \left(x^2 + \frac{3}{4} x^{4/3}\right) \Big|_{x=-8}^{x=8}$

$= 8^2 + \frac{3}{4}(8)^{4/3} - \left((-8)^2 + \frac{3}{4}(-8)^{4/3}\right)$

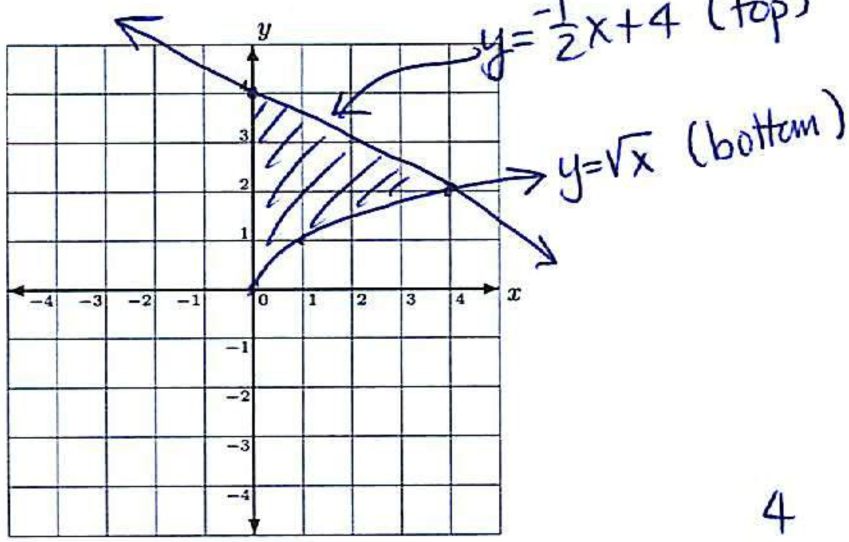
$= 64 + \frac{3}{4}(16) - \left(64 + \frac{3}{4}(16)\right)$

$= 64 + 12 - (64 + 12)$

$= 64 + 12 - 64 - 12$

$= \boxed{0}$

14. (2 pts) Find the area of the region bounded by the graphs of $y = \sqrt{x}$, $y = -\frac{1}{2}x + 4$, and $x = 0$. Be sure to first graph this region on the axis below.



$$A = \int_0^4 \left(-\frac{1}{2}x + 4 - \sqrt{x}\right) dx = \int_0^4 \left(-\frac{1}{2}x + 4 - x^{1/2}\right) dx$$

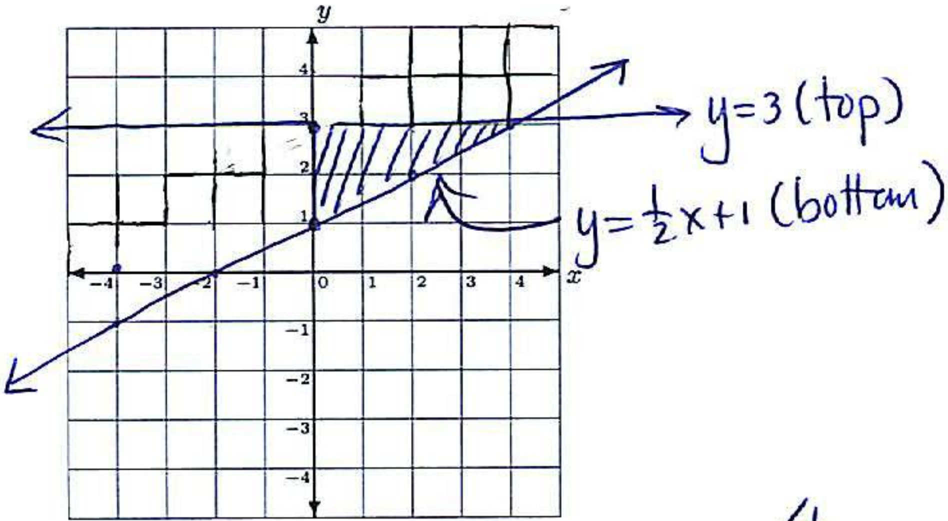
$$= \left(-\frac{1}{2}\left(\frac{1}{2}\right)x^2 + 4x + -\frac{2}{3}x^{3/2}\right) \Big|_{x=0}^{x=4}$$

$$= -\frac{1}{4}(16) + 16 - \frac{2}{3}(4)^{3/2} - 0$$

$$= -4 + 16 - \frac{2}{3}(8)$$

$$= 12 - \frac{16}{3} = \frac{36}{3} - \frac{16}{3} = \boxed{\frac{20}{3}}$$

15. (2 pts) Find the area of the region bounded by the graphs of $y = \frac{1}{2}x + 1$, $y = 3$, and $x \geq 0$. Be sure to first graph this region on the axis below.



$$A = \int_0^4 \left(3 - \frac{1}{2}x - 1\right) dx = \int_0^4 \left(2 - \frac{1}{2}x\right) dx$$

$$= \left(2x - \frac{1}{2}\left(\frac{1}{2}\right)x^2\right)\Bigg|_{x=0}^{x=4}$$

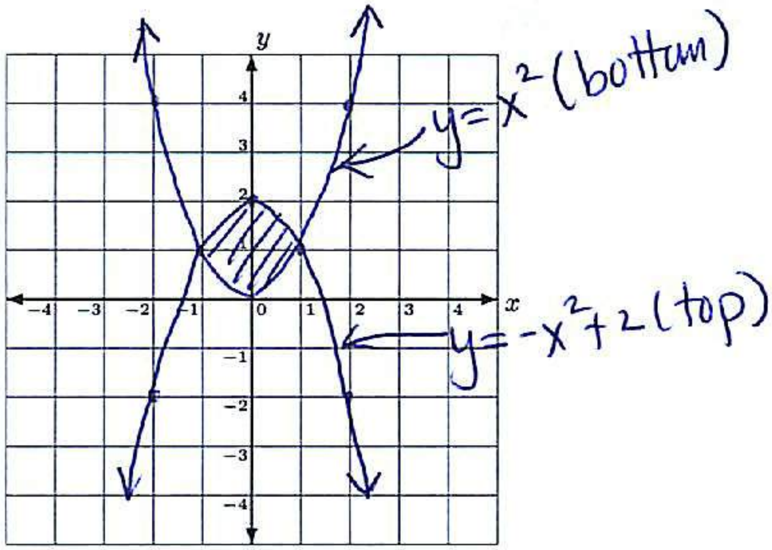
$$= 2(4) - \frac{1}{4}(4)^2 - 0$$

$$= 8 - \frac{1}{4}(16) = 8 - 4 = \boxed{4}$$

Note: area shaded is a triangle. So, could find area by

$$A = \frac{1}{2}bh = \frac{1}{2}(4)(2) = \frac{1}{2}(8) = \boxed{4}$$

16. (2 pts) Find the area of the region bounded by the graphs of $y = x^2$ and $y = -x^2 + 2$. Be sure to first graph this region on the axis below.



$$A = \int_{-1}^{1} \left(-x^2+2-x^2\right) dx = \int_{-1}^{1} \left(-2x^2+2\right) dx$$

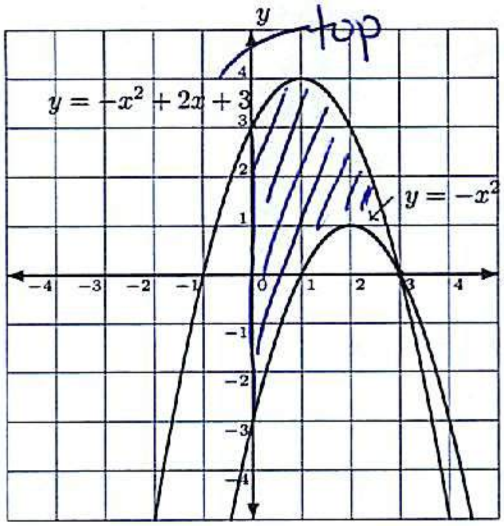
$$= \left(-\frac{2}{3}x^3+2x\right)\Big|_{x=-1}^{x=1}$$

$$= -\frac{2}{3}(1)^3+2(1)-\left(-\frac{2}{3}(-1)^3+2(-1)\right)$$

$$= -\frac{2}{3}+2-\left(\frac{2}{3}-2\right)$$

$$= -\frac{2}{3}+2-\frac{2}{3}+2 = 4-\frac{4}{3} = \frac{12}{3}-\frac{4}{3} = \boxed{\frac{8}{3}}$$

17. (1.5 pts) Find the area of the region bounded by the graphs of $y = -x^2+2x+3$, $y = -x^2+4x-3$, and $x = 0$.

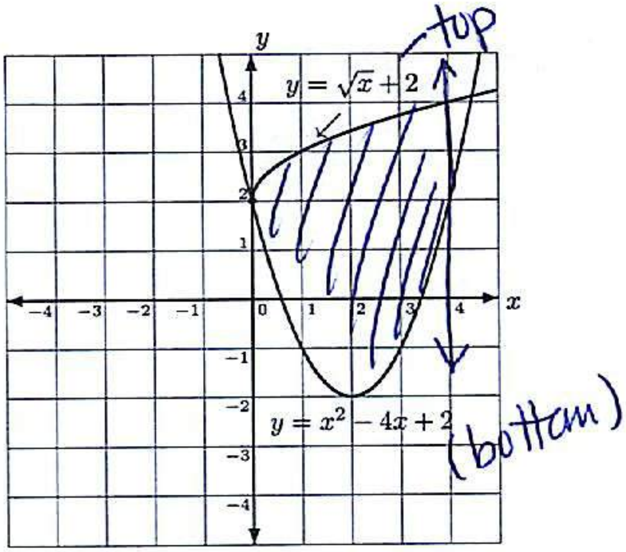


$$A = \int_0^3 \left[-x^2+2x+3 - (-x^2+4x-3)\right] dx$$

$$= \int_0^3 (-2x+6)\,dx = (-x^2+6x)\Big|_{x=0}^{x=3}$$

$$= -9+18-0 = \boxed{9}$$

18. (1.5 pts) Find the area of the region bounded by the graphs of $y = \sqrt{x}+2$, $y = x^2-4x+2$, $x = 0$, and $x = 4$.



$$A = \int_0^4 \left[\sqrt{x}+2-(x^2-4x+2)\right] dx = \int_0^4 (x^{1/2}+2-x^2+4x-2)\,dx$$

$$= \int_0^4 (x^{1/2}-x^2+4x)\,dx$$

$$= \left(\frac{2}{3}x^{3/2}-\frac{1}{3}x^3+2x^2\right)\Bigg|_{x=0}^{x=4}$$

$$= \frac{2}{3}(4)^{3/2}-\frac{1}{3}(4)^3+2(4)^2-0$$

$$= \frac{2}{3}(8)-\frac{1}{3}(64)+2(16) = \frac{16}{3}-\frac{64}{3}+32$$

$$= -\frac{48}{3}+32 = -16+32 = \boxed{16}$$