## Homework Set 8

1. (10\%) There are only two cellular telephone companies in the Los Angeles area. We can think of them as duopolists when they set per minute prices for a telephone call. Assume that consumers can change firms costlessly. The market demand function is $P=60-Q$ and marginal costs are zero.
a) What does firm 1's demand curve look like?
b) What is the Nash equilibrium in this situation?
c) How many units does each firm produce and what is the price?
d) If each form has a fixed cost of $\$ 50$, what are firm profits?
e) If the firms merged, what would be the equilibrium price? If you were an economist working for the Federal Trade Commission, would you recommend that the merger take place? Explain why or why not?
2. (10\%) Can you explain why two Bertrand competitors would build a plant with a fixed production capacity. What size would the plant be?
3. $(10 \%)$ The demand curve for a particular product is given by

$$
Q=630000-300 p .
$$

The marginal cost of producing the product is $\$ 400$. Two firms produce the product, working as a Cournot Duopoly. Plot their reaction functions. Given their reaction functions, calculate the quantity produced and the market price for the product. Note: no graph, no credit.
4. (10\%) For each of the following cases, the marginal cost of producing the product is zero. For each case, I give the demand function and the number of firms in the industry. What is the price of the product?
a) $Q=12-p ; N=2$
b) $p=36-2 Q ; N=4$
c) $Q=100-4 p ; N=3$
d) $Q=600-4 p ; N=5$
5. $(10 \%)$ The demand for a product is

$$
Q=600-2 p .
$$

The marginal cost of producing a product is zero, but each firm in the business has a fixed cost of $\$ 20$.
a) Initially two firms are producing the product in a Cournot Duopoly. How many units are being produced? At what price are they being sold? What is each firm's profit?
b) Now suppose a third firm enters the business. We know this is not technically a Cournot duopoly, but we know how to extend the model. With three firms, how many units are being produced? At what price are they being sold? What is each firm's profit?
c) These profits will be a signal to other firms to enter the business, so a fourth firm will enter. And so on. How many firms will eventually enter? When firms stop entering, what price will the product be sold for? How many firms will there be? (Hint: don't forget the fixed cost).
6. $(15 \%)$ There is a particular product produced in a duopolistic industry. The industry demand curve is

$$
Q=90-P
$$

where $Q$ is the total quantity demanded each day and $P$ is the price charged. To make life easy, we will assume that the product can be produced at zero marginal cost.. Given the peculiar nature of the industry, each firm must produce its output each night and then bring it to market the next day. The actual price is then set each day to clear the market.
a) Suppose initially that the two firms can collude and set price and production to maximize combined profits. What would be the total quantity produced? At what price would it sell? And, assuming that the firms split output 50-50, what would be the profits of each firm?
b) Now assume that the firms cannot collude or otherwise engage in cartel-like behavior. If we assume that, when all is said and done, the two forms end up with identical production and prices, there are two Nash equilibria. The first is for each firm to produce 45. Show why this is a Nash equilibrium. (And yes, you will need to define what you mean by Nash equilibrium).
c) A second Nash equilibrium occurs when each firm is producing 30. Show why that can be a Nash Equilibrium.
7. (5\%) A construction firm is negotiating with two cement firms for one of them to supply $\$ 1$ million of concrete per year over the next three years.

Discuss how the construction company should structure the contract so that it costs the construction firm the least amount over the three years.
8. (10\%) Cumberland Farms and Citco are two service stations in Morrisville Vermont, located at the same intersection. Cumberland Farms sells gas through the usual self-service routine (drive up, put your credit card in a slot, fill the tank yourself, and drive away). Citco provides "full service". They fill the tank and clean your windshield. The manager of Citco has instructions from her boss to never let her price be more than a penny a gallon above that charged by Cumberland Farms. In fact that means her price is usually exactly a penny a gallon above that of Cumberland Farms.
a. Explain why this essentially constitutes a cartel.
b. If you were the chief litigator for the Federal Trade Commission, would you attack this cartel in court? (Ignore the obvious response that you would have bigger fish to fry).
c. Draw the demand curve for gasoline from these two stations and show how Cumberland Farms should price, assuming it has figured out what is going on.
d. What would happen if the Citco manager began to match the Cumberland Farms price?
9. (5\%) In monopolistic competition, firms compete to establish a monopoly. Explain whether you agree or disagree with this statement.
10. (5\%) If members of your class tried to form a study-reduction cartel, what problems would such a cartel have?
11. (5\%) A cartel includes large and small companies, each with different long-run average and marginal cost curves. A cartel requires each member to reduce output by $15 \%$ in the short run from the total output produced. The authority assigns a quota to each firm that is $15 \%$ less than the output produced by the firm in long-run competitive equilibrium.
a) Explain why the $15 \%$ reduction rule will or will not maximize total profits of the cartel.
b) How would you assign the quota of each firm to maximize total cartel profits?
c) Can you explain why the cartel, your insight notwithstanding, might adopt the $15 \%$ rule?
12. (5\%) If a firm wants to stop bidders from colluding on a construction project, it should require written bids, and open them all at once in public. Explain whether you agree or disagree with this statement.

