Taxes and Efficiency Losses- A Proof

$$\frac{\tau_B}{\tau_A} = \frac{\left[\dfrac{R_{\tau_B=1\%}}{EL_{\tau_B=1\%}}\right]}{\left[\dfrac{R_{\tau_A=1\%}}{EL_{\tau_A=1\%}}\right]}$$

Two Propositions

$$EL_{\tau_B} \sim \tau_B^2$$

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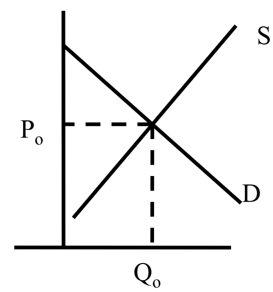
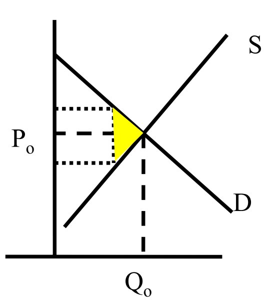
The General Efficiency Loss



The General Efficiency Loss

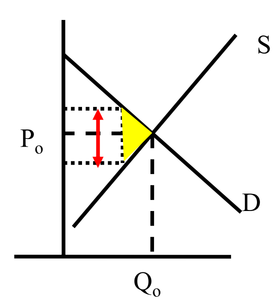
- Now suppose we impose a tax on the product, so the quantity sold drops.
- The efficiency loss is, of course, the shaded area.



The General Efficiency Loss

- The EL is

  ½BH



The General Efficiency Loss

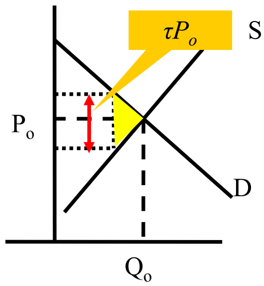
- The EL is

  $\frac{1}{2}BH$

  $B = \tau P_o$

  $H = \mu B = \mu\, \tau P_o$

  $EL = (\frac{1}{2})\, \mu\, \tau^2 P_o^2$



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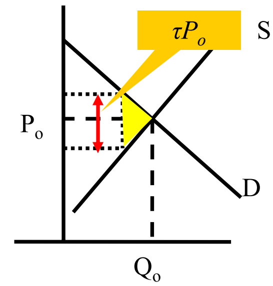

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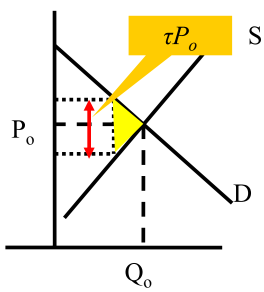

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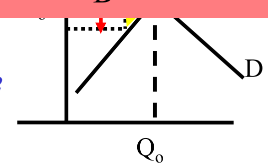

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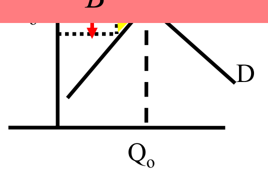
$$R_\tau = \tau R_1$$



The Second Proposition

$$EL_A = \frac{1}{2}\mu \tau_A^2 P_{oA}^2$$

$$R_\tau^A = \tau_A R_1^A$$

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$$\min\ EL_A + EL_B$$

$$\text{s.t } R_\tau^A + R_\tau^B = R^*$$

The Second Proposition

$$EL_A = \frac{1}{2}\ \frac{1}{2}\mu\tau_A^2 P_{oA}^2 + \frac{1}{2}\mu\tau_B^2 P_{oB}^2$$

$$R_\tau^A = \tau\ \ \tau_A R_1^A + \tau_B R_1^B = R^*$$

$$\min\ EL_A + EL_B$$

$$\text{s.t } R_\tau^A + R_\tau^B = R^*$$

The Algebra

$$\min \frac{1}{2}\mu\tau_A^2 P_{oA}^2 + \frac{1}{2}\mu\tau_B^2 P_{oB}^2 - \lambda(\tau_A R_1^A + \tau_B R_1^B - R^*)$$

The Algebra

$$\min \frac{1}{2}\mu\tau_A^2 P_{oA}^2 + \frac{1}{2}\mu\tau_B^2 P_{oB}^2 - \lambda(\tau_A R_1^A + \tau_B R_1^B - R^*)$$

Optimization

$$\min H = \frac{1}{2}\mu\tau_A^2 P_{oA}^2 + \frac{1}{2}\mu\tau_B^2 P_{oB}^2 - \lambda(\tau_A R_1^A + \tau_B R_1^B - R^*)$$

$$\frac{\partial H}{\partial \tau_A} = \mu\tau_A P_{oA}^2 - \lambda R_1^A = 0$$

$$\frac{\partial H}{\partial \tau_B} = \mu\tau_B P_{oB}^2 - \lambda R_1^B = 0$$

Optimization

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Optimization

$$\mu\tau_A P_{oA}^2 = \lambda R_1^A$$

$$\mu\tau_A P_{oB}^2 = \lambda R_1^B$$

$$\frac{\tau_A}{\tau_B} = \frac{R_1^A P_{oB}^2}{R_1^B P_{oA}^2}$$

Optimization

$$\frac{\tau_A}{\tau_B} = \frac{R_1^A P_{oB}^2}{R_1^B P_{oA}^2}$$

$$EL_A = \frac{1}{2}\mu\tau_A^2 P_{oA}^2$$

$$EL_{A,1} = \frac{1}{2}\mu P_{oA}^2$$

$$\frac{\tau_A}{\tau_B} = \frac{R_1^A EL_{B,1}}{R_1^B EL_{A,1}} = \left[\frac{R_1^A}{EL_{A,1}}\right]\left[\frac{R_1^B}{EL_{B,1}}\right]$$

Optimization

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End