The Cobb-Douglas Production Function

$$Y = AK^{\alpha}L^{(1-\alpha)}$$

History

- Developed by Paul Douglas and C. W. Cobb in the 1930's

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  - Douglas went on to be professor at Chicago and U.S. Senator
  - Cobb - ??

The General Problem

- An increase in a nation's capital stock or labor force means more output.
- Is there a mathematical formula that relates capital, labor and output?

The General Form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Increasing Capital

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$

$$Y = A(2K_o)^{\alpha} L_o^{1-\alpha}$$

Increasing Capital

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$

$$Y = A(2K_o)^{\alpha} L_o^{1-\alpha} =$$

$$2^{\alpha} AK_o^{\alpha} L_o^{1-\alpha} = 2^{\alpha} Y_o$$

Increasing Capital

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$

$$Y = A(2K_o)^{\alpha} L_o^{1-\alpha} =$$

$$2^{\alpha} AK_o^{\alpha} L_o^{1-\alpha} = 2^{\alpha} Y_o$$

Diminishing returns to proportion

Increasing Labor

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$

$$Y = AK_o^{\alpha} (2L_o)^{1-\alpha}$$

Increasing Labor

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$

$$Y = AK_o^{\alpha} (2L_o)^{1-\alpha} =$$

$$2^{1-\alpha} AK_o^{\alpha} L_o^{1-\alpha} = 2^{1-\alpha} Y_o$$

Increasing Labor

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$

$$Y = AK_o^{\alpha} (2L_o)^{1-\alpha} =$$

$$2^{1-\alpha} AK_o^{\alpha} L_o^{1-\alpha} = 2^{1-\alpha} Y_o$$

Diminishing returns to proportion

Increasing Both

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$

$$Y = A(2K_o)^{\alpha} (2L_o)^{1-\alpha}$$

Increasing Both

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$
$$Y = A(2K_o)^{\alpha}(2L_o)^{1-\alpha} =$$
$$2 \ AK_o^{\alpha} L_o^{1-\alpha} = 2 \ Y_o$$

Increasing Both

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$
$$Y = A(2K_o)^{\alpha}(2L_o)^{1-\alpha} =$$
$$2 \ AK_o^{\alpha} L_o^{1-\alpha} = 2 \ Y_o$$

Constant returns to ***scale***

Substitution

$$Y_o = AK_o^{\alpha} L_o^{1-\alpha}$$
$$Y = A(2K_o)^{\alpha}(xL_o)^{1-\alpha} = Y_o$$

Capital and Labor Can be Substituted

An Illustration

$$Y_t = A_t K_t^{1/2} L_t^{1/2}$$

An Illustration

$$Y = AK^{1/2} L^{1/2}$$

An Illustration

$$Y = A\sqrt{KL}$$

An Illustration

A =3

L =10

$$Y = A\sqrt{KL}$$

K =10

An Illustration

$$Y = 3\sqrt{(10)(10)} = 3\sqrt{100} = 30$$

Doubling Capital

$$Y = 3\sqrt{(20)(10)} = 3\sqrt{200} =$$

$$30\sqrt{2} \cong 42$$

Constant Returns to Scale

$$Y = 3\sqrt{(20)(20)} = 3\sqrt{400} = 60$$

Substitution

$$Y = 3\sqrt{(20)(x)} = 30$$

$$x = 5$$

Estimation

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

$$\log(Y_t) = \log(A_t) + \alpha \log(K_t) + (1-\alpha)\log(L_t)$$

Estimation

$$\log(Y_t) = C + t + \beta_1 \log(K_t) + \beta_2 \log(L_t) + \varepsilon_t$$

Estimation

$$\log(Y_t) = \alpha + \beta_1 \log(K_t) + \beta_2 \log(L_t) + \varepsilon_t$$

Statistical issues abound!

Best Estimate

$$\alpha \cong \frac{1}{3}$$

Factor Payments

- α = % of Income going to owners of capital
- $1-\alpha$ = % of Income going to workers

How well does it work?

$$Y_t = A_t K_t^{\alpha} L_t^{\beta}$$

You can't beat something with nothing

CES Production Function

$$\sigma = \frac{\%\Delta\left(\frac{K}{L}\right)}{\%\Delta\left(\frac{w}{r}\right)}$$

CES Production Function

Suppose 10% increase in wage rate leads to 10% increase in capital labor ratio. σ = 1.

$$\%\Delta\left(\frac{w}{r}\right)$$

CES Production Function

Suppose 10% increase in wage rate leads to 5% increase in capital labor ratio. σ = ½.

$$\%\Delta\left(\frac{w}{r}\right)$$

CES Production Function

In the Cobb-Douglas Production Function,

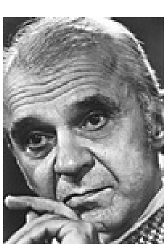
σ = 1.

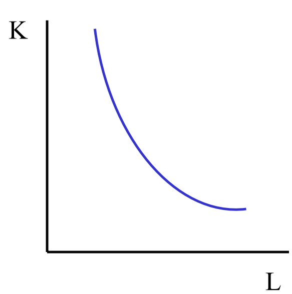
$$\%\Delta\left(\frac{w}{r}\right)$$

CES Production Function

- The CES allows for a different elasticity of substitution.
- Little gained.

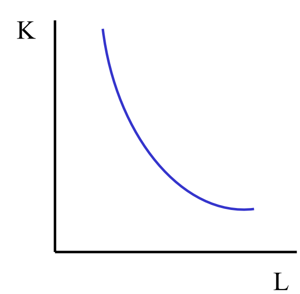
Leontief Production Function





Leontief Production Function

- K = aY
- L = bY



Leontief Production Function

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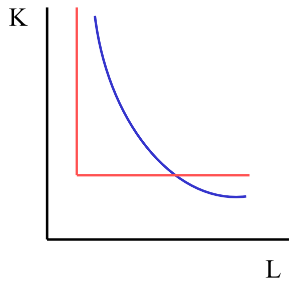
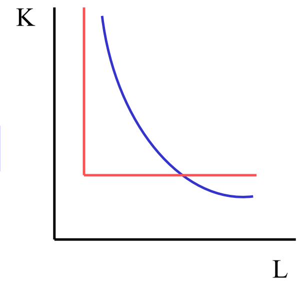


Leontief Production Function

- K = aY
- L = bY

$\sigma = 0$



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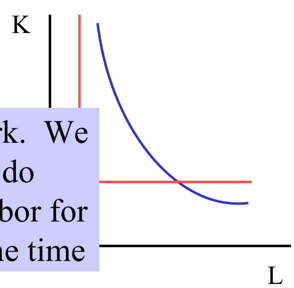

Doesn't work. We can and do substitute labor for capital all the time

Other Factors?

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} LND_t^{1-\alpha-\beta}$$

And in Conclusion…

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

End