Wealth and Taxes-The Constraint

$$z = \frac{(1-\tau_w)w_2}{(1+(1-\tau_r)r)} + \frac{(1-\tau_w)w_3}{(1+(1-\tau_r)r)^2}$$

Assumptions

- People live four periods.
- Before-tax wage income of w_1, w_2, w_3, w_4
- Before-tax interest rate of r
- Tax rates of τ_w, τ_r

The Basic Consumption Function

Our consumption function

$$c_1 = \frac{1}{4} z_1$$
$$c_2 = \frac{1}{3} z_2$$
$$c_3 = \frac{1}{2} z_3$$
$$c_4 = z_4$$

The Basic Consumption Function

Our function

$$\max \frac{1}{4}\log(c_1) + \frac{1}{3}\log(c_2) + \frac{1}{2}\log(c_3) + \log(c_4)$$

$$c_1 = \frac{1}{4} z_1$$
$$c_2 = \frac{1}{3} z_2$$
$$c_3 = \frac{1}{2} z_3$$
$$c_4 = z_4$$

Assets

$$a_1 = w_1(1-\tau_w) - c_1$$
$$a_2 = a_1[1+r(1-\tau_r)] + w_2(1-\tau_w) - c_2$$
$$a_3 = a_2[1+r(1-\tau_r)] + w_3(1-\tau_w) - c_3$$
$$a_4 = a_3[1+r(1-\tau_r)] + w_4(1-\tau_w) - c_4$$

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$a_1 = w_1(1-\tau_w) - c_1$

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$a_3 = a_2[1+r(1-\tau_r)] + w_3(1-\tau_w) - c_3$

$a_4 = a_3[1+r(1-\tau_r)] + w_4(1-\tau_w) - c_4 = 0$

Manipulation-Hold

$a_1 = w_1(1-\tau_w) - c_1$

$a_2 = a_1[1+r(1-\tau_r)] + w_2(1-\tau_w) - c_2$

$a_3 = a_2[1+r(1-\tau_r)] + w_3(1-\tau_w) - c_3$

$a_4 = a_3[1+r(1-\tau_r)] + w_4(1-\tau_w) - c_4$

Manipulation

$a_3[1+r(1-\tau_r)] + w_4(1-\tau_w) - c_4 = 0$

$a_3 = a_2[1+r(1-\tau_r)] + w_3(1-\tau_w) - c_3$

$\{ a_2[1+r(1-\tau_r)] + w_3(1-\tau_w) - c_3 \}$

$[1+r(1-\tau_r)] + w_4(1-\tau_w) - c_4 = 0$

Manipulation

$\{ a_2[1+r(1-\tau_r)] + w_3(1-\tau_w) - c_3 \}$

$[1+r(1-\tau_r)] + w_4(1-\tau_w) - c_4 = 0$

$a_2 = a_1[1+r(1-\tau_r)] + w_2(1-\tau_w) - c_2$

$\{ [a_1[1+r(1-\tau_r)] + w_2(1-\tau_w) - c_2][1+r(1-\tau_r)] + w_3(1-\tau_w) - c_3 \}$

$[1+r(1-\tau_r)] + w_4(1-\tau_w) - c_4 = 0$

Manipulation

$\{ [a_1[1+r(1- \tau_r)] + w_2(1-\tau_w) - c_2][1+r(1- \tau_r)] + w_3(1-\tau_w) - c_3\}$

$[1+r(1- \tau_r)] + w_4(1-\tau_w) - c_4 = 0$

$a_1 = w_1(1-\tau_w) - c_1$

$\{ [(w_1(1-\tau_w) - c_1)[1+r(1- \tau_r)] + w_2(1-\tau_w) - c_2][1+r(1- \tau_r)] + w_3(1-\tau_w) - c_3\}$

$[1+r(1- \tau_r)] + w_4(1-\tau_w) - c_4 = 0$

Manipulation

$\{ [(w_1(1-\tau_w) - c_1)[1+r(1- \tau_r)] + w_2(1-\tau_w) - c_2][1+r(1- \tau_r)] + w_3(1-\tau_w) - c_3\}$

$[1+r(1- \tau_r)] + w_4(1-\tau_w) - c_4 = 0$

$w_1(1-\tau_w)[1+r(1- \tau_r)]^3 + w_2(1-\tau_w)[1+r(1- \tau_r)]^2 + w_3(1-\tau_w)[1+r(1- \tau_r)] + w_4(1-\tau_w) = c_1[1+r(1- \tau_r)]^3 + c_2[1+r(1- \tau_r)]^2 + c_3[1+r(1- \tau_r)] + c_4$

Manipulation

$w_1(1-\tau_w)[1+r(1- \tau_r)]^3 + w_2(1-\tau_w)[1+r(1- \tau_r)]^2 + w_3(1-\tau_w)[1+r(1- \tau_r)] + w_4(1-\tau_w) = c_1[1+r(1- \tau_r)]^3 + c_2[1+r(1- \tau_r)]^2 + c_3[1+r(1- \tau_r)] + c_4$

Divide through by

$[1+r(1- \tau_r)]^3$

Present Value

$$c_1 + \frac{c_2}{(1+r(1-\tau_r))} + \frac{c_3}{(1+r(1-\tau_r))^2} + \frac{c_4}{(1+r(1-\tau_r))^3} =$$

$$w_1(1-\tau_w) + \frac{w_2(1-\tau_w)}{(1+r(1-\tau_r))} + \frac{w_3(1-\tau_w)}{(1+r(1-\tau_r))^2} + \frac{w_4(1-\tau_w)}{(1+r(1-\tau_r))^3}$$

Present Value

$$c_1 + \frac{c_2}{(1+r(1-\tau_r))} + \frac{c_3}{(1+r(1-\tau_r))^2} + \frac{c_4}{(1+r(1-\tau_r))^3} = z$$

$$w_1(1-\tau_w) + \frac{w_2(1-\tau_w)}{(1+r(1-\tau_r))} + \frac{w_3(1-\tau_w)}{(1+r(1-\tau_r))^2} + \frac{w_4(1-\tau_w)}{(1+r(1-\tau_r))^3}$$

The Wealth Formula

$$z = \frac{(1-\tau_w)w_2}{(1+(1-\tau_r)r)} + \frac{(1-\tau_w)w_3}{(1+(1-\tau_r)r)^2}$$

Modifications are possible

- Set up the equations for a_1, a_2, a_3, a_4
- $a_4=0$
- Add different taxes, transfer payments.
- Adjust for different interest rates and tax rates for different periods.
- Derive z.

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End

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