Minimax Strategies

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Lectures in Microeconomics-Charles W. Upton

Minimax Strategies

- Everyone who has studied a game like poker knows the importance of mixing strategies.
  - With a bad hand, you often fold
  - But you must bluff sometimes

Zero Sum Games

- Define a zero-sum game, in which one firm's profits are another firm's losses.
- Flipping coins or other betting games are straightforward examples of zero-sum games.
- Positive sum games such as buying a product are more common in economics.

Why Zero Sum Games?

- Zero sum games are easier to analyze
- They show us an important extension of game theory.

An Example

If A Follows Strategy	*If B Follows Strategy* B_1	B_2
A_1	1	2
A_2	3	1

An Example

Since this is a zero-sum game, we only display A's gains, for B's losses are exactly the opposite of A's gains

tegy	*If B Follows Strategy* B_1	B_2
	1	2
	3	1

An Example

How should B play the game?

If A Follows Strategy	If B Follows Strategy B_1	B_2
A_1	1	2
A_2	3	1

An Example

There is not a dominant strategy here.

If A Follows Strategy	If B Follows Strategy B_1	B_2
A_1	1	2
A_2	3	1

An Example

If B always follows strategy B_1, A will always follow A_2.

If B always follows strategy B_2, A will always follow A_1.

...y	If B Follows Strategy B_1	B_2
	1	2
A_2	3	1

An Example

That would suggest that A can only win $1

In fact A can do better.

...rategy	If B Follows Strategy B_1	B_2
	1	2
A_2	3	1

...mple

A mixed strategy

Suppose A follows strategy A_1 sometimes; and other times, strategy A_2.

A will always win $1 and sometimes $2 or $3, depending on what B does. Thus, it does better.

If B Follows Strategy B_1	B_2
1	2
3	1

An Example

B's Response

When B follows B_1, it loses $1 part of the time and $3 part of the time.

When it follows B_2, it loses $2 part of the time and $1 part of the time.

If B Follows Strategy B_1	B_2
1	2
3	1

An Example

B must mix strategies to minimize A's winnings

If A Follows Strategy	If B Follows Strategy B_1	B_2
A_1	1	2
A_2	3	1

An Example

Suppose

$B_1 \Rightarrow p_1$ percent of the time

$B_2 \Rightarrow (1-p_1)$ percent of the time

If A Follows Strategy	If B Follows Strategy B_1	B_2
A_1	1	2
A_2	3	1

A's Winnings

From Strategy A_1	$p_1 (1) + (1-p_1)(2)$
From Strategy A_2	$p_1 (3) + (1-p_1) (1)$

Remember, B is following strategy 1 p_1 percent of the time.

A's Winnings

	The %of time B follows B_1			
Payoff from Strategy	$p_1 = 1.0$	$p_1 = 2/3$	$p_1 = 1/3$	$p_1 = 0$
A_1	1	4/3	5/3	2
A_2	3	7/3	5/3	1

A's Winnings

Payoff from Strategy	$p_1 = 1.0$	$p_1 = 2/3$	$p_1 = 1/3$	$p_1 = 0$
A_1	1	4/3	5/3	2
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A's Winnings

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A_2	3	7/3	5/3	1

If B is following the two strategies randomly, these are A's optimal decisions

A's Winnings

	The %of time B follows B_1			
Payoff from Strategy	$p_1 = 1.0$	$p_1 = 2/3$	$p_1 = 1/3$	$p_1 = 0$
A_1	1	4/3	5/3	2
A_2	3	7/3	5/3	1

The Minimax Strategy

A's Winnings

A will follow his best strategy. B must respond by minimizing his maximum winnings.

	The %of time B follows B_1			
	$p_1 = 1.0$	$p_1 = 2/3$	$p_1 = 1/3$	$p_1 = 0$
	1	4/3	5/3	2
	3	7/3	5/3	1

A's Winnings

That means setting $p_1 = 1/3$.

	The %of time B follows B_1			
	$p_1 = 1.0$	$p_1 = 2/3$	$p_1 = 1/3$	$p_1 = 0$
A_1	1	4/3	5/3	2
A_2	3	7/3	5/3	1

A's Winnings

This is the best B can do.

It is following a strategy to minimize A's <u>maximum</u> gain.

	The %of time B follows B_1			
	.0	$p_1 = 2/3$	$p_1 = 1/3$	$p_1 = 0$
		4/3	5/3	2
		7/3	5/3	1

A's Winnings

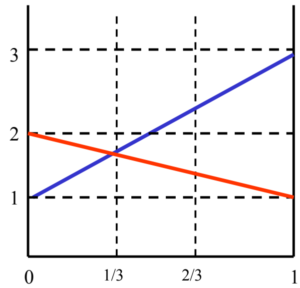
This is the *minimax* strategy

	The %of time B follows B_1			
	$p_1 = 1.0$	$p_1 = 2/3$	$p_1 = 1/3$	$p_1 = 0$
A_1	1	4/3	5/3	2
A_2	3	7/3	5/3	1

The Minimax Strategy

- There is an obvious analogy to playing poker. If you always fold a poor hand and raise a good hand, you will not make much money.
  - You must, on occasion, bet on a poor hand and fold on a good hand.
  - If not, your opponent can "read" your bets and adjust his accordingly.

The Graphical Solution



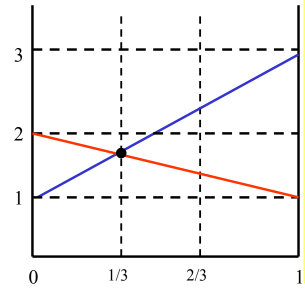
From Strategy A_1

$p_1 (1) + (1-p_1)(2)$

From Strategy A_2

$p_1 (3) + (1-p_1)(1)$

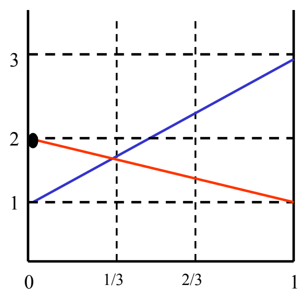
The Graphic



A's payoffs from following strategy A_1 as a function of B's probability of following B_1

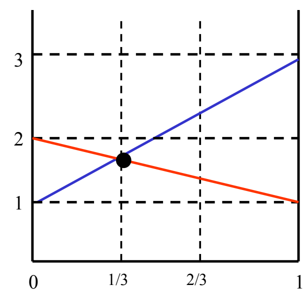
A's payoffs from following strategy A_2 as a function of B's probability of following B_1

The Graphical Solution



If $p_1 = 0$ (B never plays strategy B_1), A maximizes his winnings by playing A_1

The Graphical Solution



Given A's ability to choose strategies, B does best (or loses the least) by setting $p_1 = 1/3$

The Minimax Strategy

- Any attempt to carry this further will lead us into advanced mathematics.
- This quick introduction illustrates what can be one to set up strategy problems in a game theoretic framework.

End