

## Properties of Production Functions

$$Q = 1000\sqrt{MW}$$

## A Tabular Presentation

Units of Output from Different Combinations of Factors of Production						
Number of Machines	Number of Workers					
	1	2	3	4	5	6
1	1000	1414	1732	2000	2236	2449
2	1414	2000	2449	2828	3162	3464
3	1732	2449	3000	3464	3873	4243
4	2000	2828	3464	4000	4472	4899
5	2236	3162	3873	4472	5000	5477
6	2449	3464	4243	4899	5477	6000

## A Production Isoquant

Units of Output from Different Combinations of Factors of Production						
Number of Machines	Number of Workers					
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## Second Production Isoquant

Units of Output from Different Combinations of Factors of Production						
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## Some Points on Isoquants

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Output	Machines, Number of Workers		
1414	1,2	2,1	
1732	1,3	3,1	
2000	1,4	4,1	
2236	1,5	5,1	
2449	1,6	6,1	
3464	3,4	4,3	6,2

## Properties of Production Functions

- Constant Returns to Scale
- Diminishing Marginal Rate of Technical Substitution (MRTS)
- Diminishing Returns to Proportion

## Constant Returns to Scale

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**Double inputs; double outputs**

## Constant Returns to Scale

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Number of Machines	Number of Workers					
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**Not always true**

## Diminishing Marginal Rate of Technical Substitution (MRTS)

Some Points on Isoquants			
Output	Machines, Number of Workers		
1414	1,2	2,1	
1732	1,3	3,1	
2000	1,4	4,1	
2236	1,5	5,1	
2449	1,6	6,1	
<b>3464</b>	<b>3,4</b>	<b>4,3</b>	<b>6,2</b>

## Diminishing MRTS

I replace the first worker with one machine.

Some Points on Isoquants			
Output	Machines, Number of Workers		
1414	1,2	2,1	
1732	1,3	3,1	
2000	1,4	4,1	
2236	1,5	5,1	
2449	1,6	6,1	
<b>3464</b>	<b>3,4</b>	<b>4,3</b>	<b>6,2</b>

## Diminishing MRTS

Two machines are required to replace the second.

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Output	Machines, Number of Workers		
1414	1,2	2,1	
1732	1,3	3,1	
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## Diminishing MRTS

This is diminishing marginal rate of technical substitution (MRTS).

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1414	1,2	2,1	
1732	1,3	3,1	
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<b>3464</b>	<b>3,4</b>	<b>4,3</b>	<b>6,2</b>

## Diminishing MRTS

MRTS is like MRS.

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1414	1,2	2,1	
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2449	1,6	6,1	
<b>3464</b>	<b>3,4</b>	<b>4,3</b>	<b>6,2</b>

## Diminishing Returns to Proportion

Two machines

Diminishing Returns to Proportion						
Number of Workers	1	2	3	4	5	6
Output	1414	2000	2449	2828	3162	3464

## Diminishing Returns to Proportion

One factor fixed, one variable. A short run production function

Diminishing Returns to Proportion						
Number of Workers	1	2	3	4	5	6
Output	1414	2000	2449	2828	3162	3464

## Diminishing Returns to Proportion

Diminishing Returns to Proportion						
Number of Workers	1	2	3	4	5	6
Output	1414	2000	2449	2828	3162	3464
Output per Worker (Average Output)	1414	1000	816	707	632	581
Marginal Output per Worker	NA	586	449	379	334	302

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## Diminishing Returns to Proportion

Average product (AP) and marginal product (MP) are not the same.

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Marginal Output per Worker	NA	586	449	379	334	302

## Diminishing Returns to Proportion

MP and AP are declining.

Diminishing Returns to Proportion						
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Output	1414	2000	2449	2828	3162	3464
Output per Worker (Average Output)	1414	1000	816	707	632	581
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Properties of Production Functions

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## Diminishing Returns to Proportion

They may initially rise but must eventually decline.

Diminishing Returns to Proportion						
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Output	1414	2000	2449	2828	3162	3464
Output per Worker (Average Output)	1414	1000	816	707	632	581
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Properties of Production Functions

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## Diminishing Returns to Proportion

Marginal product can be zero but not negative

Diminishing Returns to Proportion						
Number of Workers	1	2	3	4	5	6
Output	1414	2000	2449	2828	3162	3464
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Properties of Production Functions

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## Summary

- Constant Returns to Scale
  - Sometimes, but not always

Properties of Production Functions

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## Summary

- Constant Returns to Scale
  - Sometimes, but not always
- Diminishing MRTS
  - Always

Properties of Production Functions

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## Summary

- Constant Returns to Scale
  - Sometimes, but not always
- Diminishing MRTS
  - Always
- Diminishing Returns to Proportion
  - AP and MP eventually *must* decline.

Properties of Production Functions

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## The Final Word

K

Our production isoquants are very smooth; substitution is easy (but diminishing MRTS).

While substitution may not always be that smooth, economists would always stress the possibilities of substitution.

L

## The Final Word

K

- Substitution possibilities abound
- There is more than one way to
  - Skin a cat
  - Bake a cake
  - Make a widget

L

End

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