PART SIX

Inventory Management and Scheduling

The chapters in this part relate to the management and control of inventories, and scheduling, often key factors in the success or failure of operations management to achieve profit and/or cost objectives while satisfying customers. The basic issues are how to best manage resources to effectively match supply and demand.

The chapters in this part cover the following topics:

CHAPTER 12

Inventory Management

LEARNING OBJECTIVES

After completing this chapter, you should be able to:

1 Define the term *inventory,* list the major reasons for holding inventories, and list the main requirements for effective inventory management.
2 Discuss the nature and importance of service inventories.
3 Discuss periodic and perpetual review systems.
4 Discuss the objectives of inventory management.
5 Describe the A-B-C approach and explain how it is useful.
6 Describe the basic EOQ model and its assumptions and solve typical problems.
7 Describe the economic production quantity model and solve typical problems.
8 Describe the quantity discount model and solve typical problems.
9 Describe reorder point models and solve typical problems.
10 Describe situations in which the single-period model would be appropriate, and solve typical problems.

CHAPTER OUTLINE

Inventory management is a core operations management activity. Good inventory management is important for the successful operation of most businesses and their supply chains. Operations, marketing, and finance have interests in good inventory management. Poor inventory management hampers operations, diminishes customer satisfaction, and increases operating costs.

Some organizations have excellent inventory management, and many have satisfactory inventory management. Too many, however, have unsatisfactory inventory management, which sometimes is a sign that management does not recognize the importance of inventories. More often than not, though, the recognition is there. What is lacking is an understanding of what needs to be done and how to do it. This chapter presents the concepts that underlie good inventory management. Topics include functions of inventories, requirements for effective inventory management, objectives of inventory control, and techniques for determining *how much* to order and *when* to order.

INTRODUCTION

An **inventory** is a stock or store of goods. Firms typically stock hundreds or even thousands of items in inventory, ranging from small things such as pencils, paper clips, screws, nuts, and bolts to large items such as machines, trucks, construction equipment, and airplanes. Naturally, many of the items a firm carries in inventory relate to the kind of business it engages in. Thus, manufacturing firms carry supplies of raw materials, purchased parts, partially finished items, and finished goods, as well as spare parts for machines, tools, and other supplies. Department stores carry clothing, furniture, carpeting, stationery, cosmetics, gifts, cards, and toys. Some also stock sporting goods, paints, and tools. Hospitals stock drugs, surgical supplies, life-monitoring equipment, sheets and pillow cases, and more. Supermarkets stock fresh and canned foods, packaged and frozen foods, household supplies, magazines, baked goods, dairy products, produce, and other items.

Inventory A stock or store of goods.

The inventory models described in this chapter relate primarily to what are referred to as *independent-demand* items, that is, items that are ready to be sold or used. Chapter 13 describes models that are used for *dependent-demand* items, which are components of finished products, rather than the finished products themselves. Thus, a computer would be an independent-demand item, while the components that are used to assemble a computer would be dependent-demand items: The demand for those items would depend on how many of each item is needed for a computer, as well as how many computers are going to be made.

$$$ **NEWSCLIP**

We proceed as follows. First look for a five-by-five-by-three-foot bin of gears or parts that looks like it has been there awhile. Pick up a gear and ask, casually, "How much is this worth?" You then ask, "How many of these are in the bin?" followed by, "How long has this bin been here?" and, "What's your cost of money for this company?" I recall one case in a nameless South American country where the unit cost times the number of parts times the time it had been there times the interest rate resulted in a cost-per-day figure that would insure comfortable retirement for the plant manager on the bank of the Rio de la Plata at one of the better resorts to be found there. The plant manager suddenly realized that what he was holding was not just a chunk of high-test steel, but was *real money.* He then pointed out that *he* now understood the value of the inventory but could I suggest a way to drive the point home to upper management? I suggested that he go to the accounting department and borrow enough money to be equal to the bin's value for as long as it had been sitting there, and pile it on the top of the bin. I further suggested that he do that for every bin on the production line. We rapidly figured out that by the time we had the money piled up on the bin, you would not even be able to *see* the bin. My opinion was that if the upper managers were given a tour of the line with the money piled up, they would *never* forget it.

Source: Gene Woolsey, "On Doing Good Things and Dumb Things in Production and Inventory Control," *Interfaces* 5, no. 3 (May 1975). Copyright 1975, The Institute of Management Science. Reprinted by permission.

THE NATURE AND IMPORTANCE OF INVENTORIES

Inventories are a vital part of business. Not only are they necessary for operations, but they also contribute to customer satisfaction. To get a sense of the significance of inventories, consider the following: Some very large firms have tremendous amounts of inventory. For example, General Motors was at one point reported to have as much as $40 billion worth of materials, parts, cars, and trucks in its supply chain! Although the amounts and dollar values of inventories carried by different types of firms vary widely, a typical firm probably has about 30 percent of its current assets and perhaps as much as 90 percent of its working capital invested in inventory. One widely used measure of managerial performance relates to *return on investment* (ROI), which is profit after taxes divided by total assets. Because inventories may represent a significant portion of total assets, a reduction of inventories can result in a significant increase in ROI. Furthermore, the ratio of inventories to sales in the manufacturing, wholesale, and retail sectors is one measure that is used to gauge the health of the U.S. economy.

Inventory decisions in service organizations can be especially critical. Hospitals, for example, carry an array of drugs and blood supplies that might be needed on short notice. Being out of stock on some of these could imperil the well-being of a patient. However, many of these items have a limited shelf life, so carrying large quantities would mean having to dispose of unused, costly supplies. On-site repair services for computers, printers, copiers, and fax machines also have to carefully consider which parts to bring to the site to avoid having to make an extra trip to obtain parts. The same goes for home repair services such as electricians, appliance repairers, and plumbers.

The major source of revenues for retail and wholesale businesses is the sale of merchandise (i.e., inventory). In fact, in terms of dollars, the inventory of goods held for sale is one of the largest assets of a merchandising business. Retail stores that sell clothing wrestle with decisions about which styles to carry, and how much of each to stock, knowing full well that fast-selling items will mean greater profits than having to heavily discount goods that didn't sell.

The different kinds of inventories include the following:

Raw materials and purchased parts.

Partially completed goods, called *work-in-process (WIP)*.

Finished-goods inventories (manufacturing firms) or merchandise (retail stores).

Replacement parts, tools, and supplies.

Goods-in-transit to warehouses or customers (pipeline inventory).

Both manufacturing and service organizations have to take into consideration the space requirements of inventory. In some cases, space limitations may pose restrictions on inventory storage capability, thereby adding another dimension to inventory decisions.

To understand why firms have inventories at all, you need to be aware of the various functions of inventory.

Functions of Inventory

Inventories serve a number of functions. Among the most important are the following:

1. *To meet anticipated customer demand.* A customer can be a person who walks in off the street to buy a new stereo system, a mechanic who requests a tool at a tool crib, or a manufacturing operation. These inventories are referred to as *anticipation stocks* because they are held to satisfy expected (i.e., *average*) demand.

2. *To smooth production requirements.* Firms that experience seasonal patterns in demand often build up inventories during preseason periods to meet overly high requirements during seasonal periods. These inventories are aptly named *seasonal inventories.* Companies that process fresh fruits and vegetables deal with seasonal inventories. So do stores that sell greeting cards, skis, snowmobiles, or Christmas trees.

3. *To decouple operations.* Historically, manufacturing firms have used inventories as buffers between successive operations to maintain continuity of production that would otherwise be disrupted by events such as breakdowns of equipment and accidents that cause a portion of the operation to shut down temporarily. The buffers permit other operations to continue temporarily while the problem is resolved. Similarly, firms have used buffers of raw materials to insulate production from disruptions in deliveries from suppliers, and finished goods inventory to buffer sales operations from manufacturing disruptions. More recently, companies have taken a closer look at buffer inventories, recognizing the cost and space they require, and realizing that finding and eliminating sources of disruptions can greatly decrease the need for decoupling operations.

Inventory buffers are also important in supply chains. Careful analysis can reveal both points where buffers would be most useful and points where they would merely increase costs without adding value.

4. *To protect against stockouts.* Delayed deliveries and unexpected increases in demand increase the risk of shortages. Delays can occur because of weather conditions, supplier stockouts, deliveries of wrong materials, quality problems, and so on. The risk of shortages can be reduced by holding *safety stocks,* which are stocks in excess of average demand to compensate for *variabilities* in demand and lead time.

5. *To take advantage of order cycles.* To minimize purchasing and inventory costs, a firm often buys in quantities that exceed immediate requirements. This necessitates storing some or all of the purchased amount for later use. Similarly, it is usually economical to produce in large rather than small quantities. Again, the excess output must be stored for later use. Thus, inventory storage enables a firm to buy and produce in *economic lot sizes* without having to try to match purchases or production with demand requirements in the short run. This results in *periodic* orders, or order *cycles.* The resulting stock is known as *cycle stock.* Order cycles are not always based on economic lot sizes. In some instances, it is practical or economical to group orders and/or to order at fixed intervals.

6. *To hedge against price increases.* Occasionally a firm will suspect that a substantial price increase is about to occur and purchase larger-than-normal amounts to beat the increase. The ability to store extra goods also allows a firm to take advantage of price discounts for larger orders.

Little's Law The average amount of inventory in a system is equal to the product of the average demand rate and the average time a unit is in the system.

7. *To permit operations.* The fact that production operations take a certain amount of time (i.e., they are not instantaneous) means that there will generally be some work-in-process inventory. In addition, intermediate stocking of goods—including raw materials, semifinished items, and finished goods at production sites, as well as goods stored in warehouses—leads to *pipeline* inventories throughout a production-distribution system. **Little's Law** can be useful in quantifying pipeline inventory. It states that the average amount of inventory in a system is equal to the product of the average rate at which inventory units leave the system (i.e., the average demand rate) and the average time a unit is in the system. Thus, if a unit is in the system for an average of 10 days, and the demand rate is 5 units per day, the average inventory is 50 units: 5 units/day $\times$ 10 days $=$ 50 units.

8. *To take advantage of quantity discounts.* Suppliers may give discounts on large orders.

Objectives of Inventory Control

Inadequate control of inventories can result in both under- and overstocking of items. Understocking results in missed deliveries, lost sales, dissatisfied customers, and production bottlenecks; overstocking unnecessarily ties up funds that might be more productive elsewhere. Although overstocking may appear to be the lesser of the two evils, the price tag for excessive overstocking can be staggering when inventory holding costs are high—as illustrated by the little story about the bin of gears at the beginning of the chapter—and matters can easily get out of hand. It is not unheard of for managers to discover that their firm has a 10-year supply of some item. (No doubt the firm got a good buy on it!)

Inventory management has two main concerns. One is the *level of customer service,* that is, to have the right goods, in sufficient quantities, in the right place, at the right time. The other is the *costs of ordering and carrying inventories.*

The overall objective of inventory management is to achieve satisfactory levels of customer service while keeping inventory costs within reasonable bounds. Toward this end, the decision maker tries to achieve a balance in stocking. He or she must make two fundamental decisions: the *timing* and *size* of orders (i.e., when to order and how much to order). The greater part of this chapter is devoted to models that can be applied to assist in making those decisions.

Inventory turnover Ratio of average cost of goods sold to average inventory investment.

Managers have a number of measures of performance they can use to judge the effectiveness of inventory management. The most obvious, of course, is customer satisfaction, which they might measure by the number and quantity of backorders and/or customer complaints. A widely used measure is **inventory turnover,** which is the ratio of annual cost of goods sold to average inventory investment. The turnover ratio indicates how many times a year the inventory is sold. Generally, the higher the ratio, the better, because that implies more efficient use of inventories. However, the desirable number of turns depends on the industry and what the profit margins are. The higher the profit margins, the lower the acceptable number of inventory turns, and vice versa. Also, a product that takes a long time to manufacture, or a long time to sell, will have a low turnover rate. This is often the case with high-end retailers (high profit margins). Conversely, supermarkets (low profit margins) have a fairly high turnover rate. Note, though, that there should be a balance between inventory investment and maintaining good customer service. Managers often use inventory turnover to evaluate inventory management performance; monitoring this metric over time can yield insights into changes in performance.

Another useful measure is days of inventory on hand, a number that indicates the expected number of days of sales that can be supplied from existing inventory. Here, a balance is desirable; a high number of days might imply excess inventory, while a low number might imply a risk of running out of stock.

REQUIREMENTS FOR EFFECTIVE INVENTORY MANAGEMENT

Management has two basic functions concerning inventory. One is to establish a system of keeping track of items in inventory, and the other is to make decisions about how much and when to order. To be effective, management must have the following:

1. A system to *keep track of the inventory* on hand and on order.
2. A reliable *forecast of demand* that includes an indication of possible *forecast error.*
3. Knowledge of *lead times* and *lead time variability.*
4. Reasonable estimates of inventory *holding costs, ordering costs,* and *shortage costs.*
5. A *classification system* for inventory items.

Let's take a closer look at each of these requirements.

Inventory Counting Systems

Inventory counting systems can be periodic or perpetual. Under a **periodic system,** a physical count of items in inventory is made at periodic intervals (e.g., weekly, monthly) in order to decide how much to order of each item. Many small retailers use this approach: A manager periodically checks the shelves and stockroom to determine the quantity on hand. Then the manager estimates how much will be demanded prior to the next delivery period and bases the order quantity on that information. An advantage of this type of system is that orders for many items occur at the same time, which can result in economies in processing and shipping orders. There are also several disadvantages of periodic reviews. One is a lack of control between reviews. Another is the need to protect against shortages between review periods by carrying extra stock.

Periodic system Physical count of items in inventory made at periodic intervals (weekly, monthly).

A **perpetual inventory system** (also known as a *continual* system) keeps track of removals from inventory on a continuous basis, so the system can provide information on the current level of inventory for each item. When the amount on hand reaches a predetermined minimum, a fixed quantity, Q, is ordered. An obvious advantage of this system is the control provided by the continuous monitoring of inventory withdrawals. Another advantage is the fixed-order quantity; management can determine an optimal order quantity. One disadvantage of this approach is the added cost of record keeping. Moreover, a physical count of inventories must still be performed periodically to verify records because of possible errors, pilferage, spoilage, and other factors that can reduce the effective amount of inventory. Bank transactions such as customer deposits and withdrawals are examples of continuous recording of inventory changes.

Perpetual inventory system System that keeps track of removals from inventory continuously, thus monitoring current levels of each item.

Perpetual systems range from very simple to very sophisticated. A **two-bin system,** a very elementary system, uses two containers for inventory. Items are withdrawn from the first bin until its contents are exhausted. It is then time to reorder. Sometimes an order card is placed at the bottom of the first bin. The second bin contains enough stock to satisfy expected demand until the order is filled, plus an extra cushion of stock that will reduce the chance of a stockout if the order is late or if usage is greater than expected. The advantage of this system is that there is no need to record each withdrawal from inventory; the disadvantage is that the reorder card may not be turned in for a variety of reasons (e.g., misplaced, the person responsible forgets to turn it in).

Two-bin system Two containers of inventory; reorder when the first is empty.

A technician uses a handheld Metrologic Bar Code Scanner to scan computer components prior to installation at the 300,000-square-foot Dell server manufacturing plant in Round Rock, Texas. Plastic bins are filled with parts—hard drives, processors, memory—to build computers that have already beeen ordered.

www.dell.com

Perpetual systems can be either *batch* or *online.* In batch systems, inventory records are collected periodically and entered into the system. In online systems, the transactions are recorded immediately. The advantage of online systems is that they are always up-to-date. In batch systems, a sudden surge in demand could result in reducing the amount of inventory below the reorder point between the periodic read-ins. Frequent batch collections can minimize that problem.

Universal product code (UPC) Bar code printed on a label that has information about the item to which it is attached.

Supermarkets, discount stores, and department stores have always been major users of periodic counting systems. Today, most have switched to computerized checkout systems using a laser scanning device that reads a **universal product code (UPC),** or *bar code,* printed on an item tag or on packaging. A typical grocery product code is illustrated here.

0 14800 23208

The zero on the left of the bar code identifies this as a grocery item, the first five numbers (14800) indicate the manufacturer (Mott's), and the last five numbers (23208) indicate the specific item (natural-style applesauce). Items in small packages, such as candy and gum, use a six-digit number.

UPC scanners represent major benefits to supermarkets. In addition to their increase in speed and accuracy, these systems give managers continuous information on inventories, reduce the need for periodic inventories and order-size determinations, and improve the level of customer service by indicating the price and quantity of each item on the customer's receipt, as in the following illustration:

BRACO CAPELLINI	.79
BUB YUM DBL LIME	.30 T
2/LO FAT MILK H G	1.03
EUROP ROLLS	.91
HUNTS TOMATO	.55
NEWSPAPER	.35
KR CAS BRICK CHEES	1.59
GRAPES-GREEN	
.91 LB @ .89 PER LB	.81
TAX DUE	.02
TOTAL	6.35
CASH	20.00*
CHANGE	13.65
8/07/06 18:01 21 16 23100 2570	

Bar coding is important for other sectors of business besides retailing. Manufacturing and service industries benefit from the simplified production and inventory control it provides. In manufacturing, bar codes attached to parts, subassemblies, and finished goods greatly facilitate counting and monitoring activities. Automatic routing, scheduling, sorting, and packaging can also be done using bar codes. In health care, use of bar codes can help to reduce drug dispensing errors.

Radio frequency identification (RFID) tags are also used to keep track of inventory in certain applications.

Radio Frequency Identification (RFID) Tags

READING

Keeping track of inventories in-house and throughout a supply chain is vitally important for manufacturing, service, and retail operations. Bar codes have long been used for that purpose, but they carry only a limited amount of information and require direct line-of-sight to be scanned. Radio frequency identification (RFID) tags are a technological breakthrough in inventory management, providing real-time information that increases the ability to track and process shipping containers, parts in warehouses, items on supermarket shelves, and a whole lot more. They carry much more information than bar codes, and they don't require line-of-sight to be scanned.

RFID tags transmit product information or other data to network-connected RFID readers via radio waves. Tags attached to pallets, boxes, or individual items can enable a business to identify, track, monitor, or locate any object that is within range of a reader. For example, the tags are used in "speed passes" for toll roads.

In agriculture, fruit growers might use RFID tags to constantly monitor temperatures around fruit during shipping. This ensures that the fruit is kept at appropriate temperature. The tags can be used for a wide range of agricultural products, containing information such as cultivation history, as well as whether the fruit is organically grown and what fertilizers or chemicals have been used.

Because major retail chains, such as Wal-Mart and Target, and governmental agencies now require their suppliers to use RFID tags, many companies have already made RFID a priority in their business strategies.

Although RFID technology holds the potential for improved safety, convenience, and inventory management, widespread adoption, particularly in retail operations, could take several years. Until a global standard is established and cheap disposable tags are developed, the main areas of growth continue to be in nonretail operations.

Speedpass uses RFID technology at Exxon and Mobil service stations.

www.speedpass.com

Demand Forecasts and Lead-Time Information

Inventories are used to satisfy demand requirements, so it is essential to have reliable estimates of the amount and timing of demand. Similarly, it is essential to know how long it will take for orders to be delivered. In addition, managers need to know the extent to which demand and **lead time** (the time between submitting an order and receiving it) might vary; the greater the potential variability, the greater the need for additional stock to reduce the risk of a shortage between deliveries. Thus, there is a crucial link between forecasting and inventory management.

Lead time Time interval between ordering and receiving the order.

Point-of-sale (POS) systems electronically record actual sales. Knowledge of actual sales can greatly enhance forecasting and inventory management: By relaying information about actual demand in real time, these systems enable management to make any necessary changes to restocking decisions. These systems are being increasingly emphasized as an important input to effective supply chain management by making this information available to suppliers.

Point-of-sale (POS) systems Record items at time of sale.

Inventory Costs

Three basic costs are associated with inventories: holding, transaction (ordering), and shortage costs.

Holding, or **carrying, costs** relate to physically having items in storage. Costs include interest, insurance, taxes (in some states), depreciation, obsolescence, deterioration, spoilage, pilferage, breakage, and warehousing costs (heat, light, rent, security). They also include opportunity costs associated with having funds that could be used elsewhere tied up in inventory. Note that it is the *variable* portion of these costs that is pertinent.

Holding (carrying) cost Cost to carry an item in inventory for a length of time, usually a year.

The significance of the various components of holding cost depends on the type of item involved, although taxes, interest, and insurance are generally based on the dollar value of an inventory. Items that are easily concealed (e.g., pocket cameras, transistor radios, calculators) or fairly expensive (cars, TVs) are prone to theft. Fresh seafood, meats and poultry, produce, and baked goods are subject to rapid deterioration and spoilage. Dairy products, salad dressings, medicines, batteries, and film also have limited shelf lives.

Holding costs are stated in either of two ways: as a percentage of unit price or as a dollar amount per unit. Typical annual holding costs range from 20 percent to 40 percent of the value of an item. In other words, to hold a $100 item in inventory for one year could cost from $20 to $40.

Ordering costs Costs of ordering and receiving inventory.

Ordering costs are the costs of ordering and receiving inventory. They are the costs that vary with the actual placement of an order. Besides shipping costs, they include determining how much is needed, preparing invoices, shipping costs, inspecting goods upon arrival for quality and quantity, and moving the goods to temporary storage. Ordering costs are generally expressed as a fixed dollar amount per order, regardless of order size.

When a firm produces its own inventory instead of ordering it from a supplier, the costs of machine setup (e.g., preparing equipment for the job by adjusting the machine, changing cutting tools) are analogous to ordering costs; that is, they are expressed as a fixed charge per production run, regardless of the size of the run.

Shortage costs Costs resulting when demand exceeds the supply of inventory; often unrealized profit per unit.

Shortage costs result when demand exceeds the supply of inventory on hand. These costs can include the opportunity cost of not making a sale, loss of customer goodwill, late charges, and similar costs. Furthermore, if the shortage occurs in an item carried for internal use (e.g., to supply an assembly line), the cost of lost production or downtime is considered a shortage cost. Such costs can easily run into hundreds of dollars a minute or more. Shortage costs are sometimes difficult to measure, and they may be subjectively estimated.

Classification System

An important aspect of inventory management is that items held in inventory are not of equal importance in terms of dollars invested, profit potential, sales or usage volume, or stockout penalties. For instance, a producer of electrical equipment might have electric generators, coils of wire, and miscellaneous nuts and bolts among the items carried in inventory. It would be unrealistic to devote equal attention to each of these items. Instead, a more reasonable approach would be to allocate control efforts according to the *relative importance* of various items in inventory.

A-B-C approach Classifying inventory according to some measure of importance, and allocating control efforts accordingly.

The **A-B-C approach** classifies inventory items according to some measure of importance, usually annual dollar value (i.e., dollar value per unit multiplied by annual usage rate), and then allocates control efforts accordingly. Typically, three classes of items are used: A (very important), B (moderately important), and C (least important). However, the actual number of categories may vary from organization to organization, depending on the extent to which a firm wants to differentiate control efforts. With three classes of items, A items generally account for about 10 to 20 percent of the *number* of items in inventory but about 60 to 70 percent of the *annual dollar value.* At the other end of the scale, C items might

A gas lineman for a public works company is processing data on a worksite in Ventura, California. Entering and processing data on site allows for more accurate, up-to-date records of costs as well as inventory for repairs and replacement of gas lines.

account for about 50 to 60 percent of the number of items but only about 10 to 15 percent of the dollar value of an inventory. These percentages vary from firm to firm, but in most instances a relatively small number of items will account for a large share of the value or cost associated with an inventory, and these items should receive a relatively greater share of control efforts. For instance, A items should receive close attention through frequent reviews of amounts on hand and control over withdrawals, where possible, to make sure that customer service levels are attained. The C items should receive only loose control (two-bin system, bulk orders), and the B items should have controls that lie between the two extremes.

Note that C items are not necessarily *un*important; incurring a stockout of C items such as the nuts and bolts used to assemble manufactured goods can result in a costly shutdown of an assembly line. However, due to the low annual dollar value of C items, there may not be much additional cost incurred by ordering larger quantities of some items, or ordering them a bit earlier.

EXAMPLE 1

The annual dollar value of 12 items has been calculated according to annual demand and unit cost. The annual dollar values were then arrayed from highest to lowest to simplify classification of items. (Notice that the item numbers are not in what would have been their original sequence due to arraying by annual dollar values.)

Item Number	Annual Demand	×	Unit Cost	=	Annual Dollar Value	Classification
8	1,000		$4,000		$ 4,000,000	**A**
5	3,900		700		2,730,000	**A**
3	1,900		500		950,000	**B**
6	1,000		915		915,000	**B**
1	2,500		330		825,000	**B**
4	1,500		100		150,000	**C**
12	400		300		120,000	**C**
11	500		200		100,000	**C**
9	8,000		10		80,000	**C**
2	1,000		70		70,000	**C**
7	200		210		42,000	**C**
10	9,000		2		18,000	**C**
					10,000,000	

The first two items have a relatively high annual dollar value, so it seems reasonable to classify them as A items. The next three items appear to have moderate annual dollar values and should be classified as B items. The remainder are C items, because of their relatively low annual dollar value.

Although annual dollar value may be the primary factor in classifying inventory items, a manager may take other factors into account in making exceptions for certain items (e.g., changing the classification of a B item to an A item). Factors may include the risk of obsolescence, the risk of a stockout, the distance of a supplier, and so on.

Figure 12.1 illustrates the A-B-C concept.

Managers use the A-B-C concept in many different settings to improve operations. One key use occurs in customer service, where a manager can focus attention on the most important aspects of customer service by categorizing different aspects as very important, important, or of only minor importance. The point is to not overemphasize minor aspects of customer service at the expense of major aspects.

Another application of the A-B-C concept is as a guide to **cycle counting,** which is a physical count of items in inventory. The purpose of cycle counting is to reduce discrepancies between the amounts indicated by inventory records and the actual quantities of inventory on hand. Accuracy is important because inaccurate records can lead to disruptions in operations, poor customer service, and unnecessarily high inventory carrying costs.

Cycle counting A physical count of items in inventory.

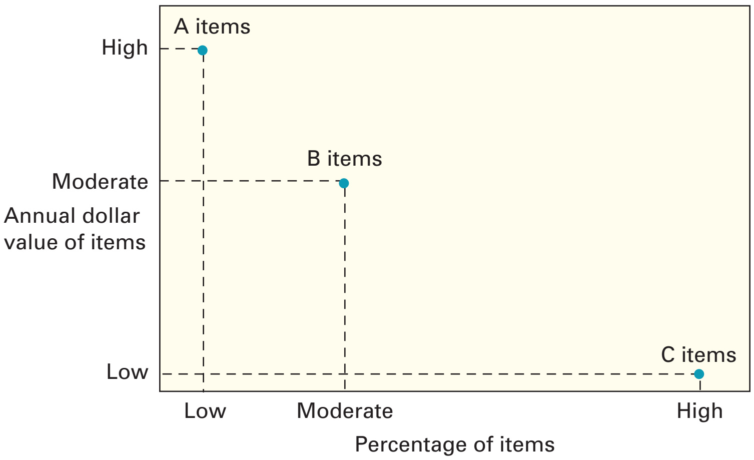


FIGURE 12.1
A typical A-B-C breakdown in relative annual dollar value of items and number of items by category

The counts are conducted more frequently than once a year, which reduces the costs of inaccuracies compared to only doing an annual count, by allowing for investigation and correction of the causes of inaccuracies.

The key questions concerning cycle counting for management are

1. How much accuracy is needed?
2. When should cycle counting be performed?
3. Who should do it?

APICS recommends the following guidelines for inventory record accuracy: ± 0.2 percent for A items, ± 1 percent for B items, and ± 5 percent for C items. A items are counted frequently, B items are counted less frequently, and C items are counted the least frequently.

Some companies use certain events to trigger cycle counting, whereas others do it on a periodic (scheduled) basis. Events that can trigger a physical count of inventory include an out-of-stock report written on an item indicated by inventory records to be in stock, an inventory report that indicates a low or zero balance of an item, and a specified level of activity (e.g., every 2,000 units sold).

Some companies use regular stockroom personnel to do cycle counting during periods of slow activity while others contract with outside firms to do it on a periodic basis. Use of an outside firm provides an independent check on inventory and may reduce the risk of problems created by dishonest employees. Still other firms maintain full-time personnel to do cycle counting.

HOW MUCH TO ORDER: ECONOMIC ORDER QUANTITY MODELS

Economic order quantity (EOQ) The order size that minimizes total annual cost.

The question of how much to order is frequently determined by using an **economic order quantity (EOQ)** model. EOQ models identify the optimal order quantity by minimizing the sum of certain annual costs that vary with order size. Three order size models are described here:

1. The basic economic order quantity model.
2. The economic production quantity model.
3. The quantity discount model.

Basic Economic Order Quantity (EOQ) Model

The basic EOQ model is the simplest of the three models. It is used to identify a *fixed* order size that will minimize the sum of the annual costs of holding inventory and ordering inventory. The unit purchase price of items in inventory is not generally included in the total cost because the unit cost is unaffected by the order size unless quantity discounts are a factor.

1. Only one product is involved.
2. Annual demand requirements are known.
3. Demand is spread evenly throughout the year so that the demand rate is reasonably constant.
4. Lead time does not vary.
5. Each order is received in a single delivery.
6. There are no quantity discounts.

TABLE 12.1
Assumptions of the basic EOQ model

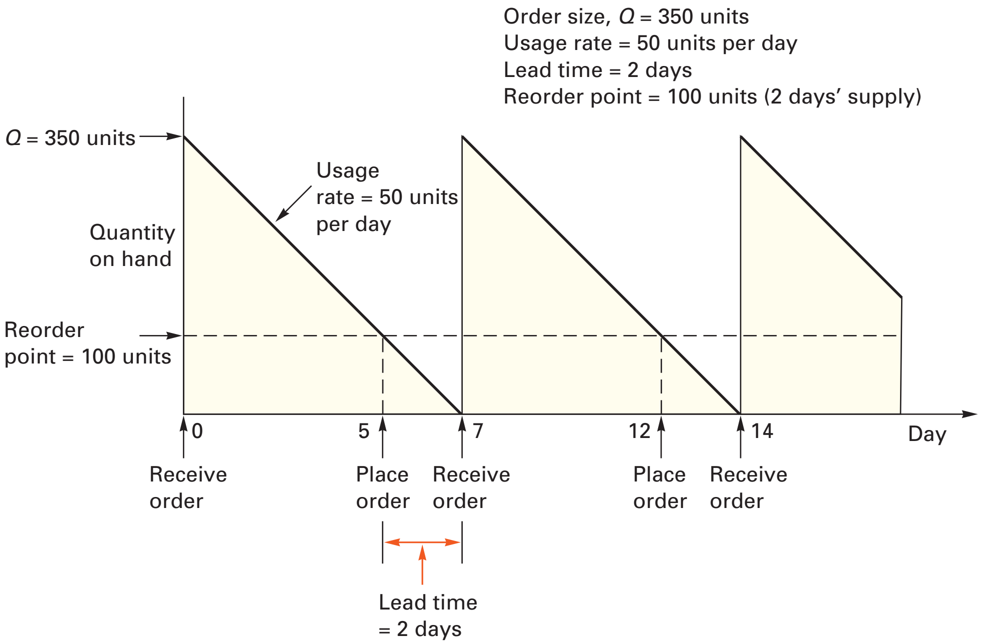


FIGURE 12.2
The inventory cycle: profile of inventory level over time

If holding costs are specified as a percentage of unit cost, then unit cost is indirectly included in the total cost as a part of holding costs.

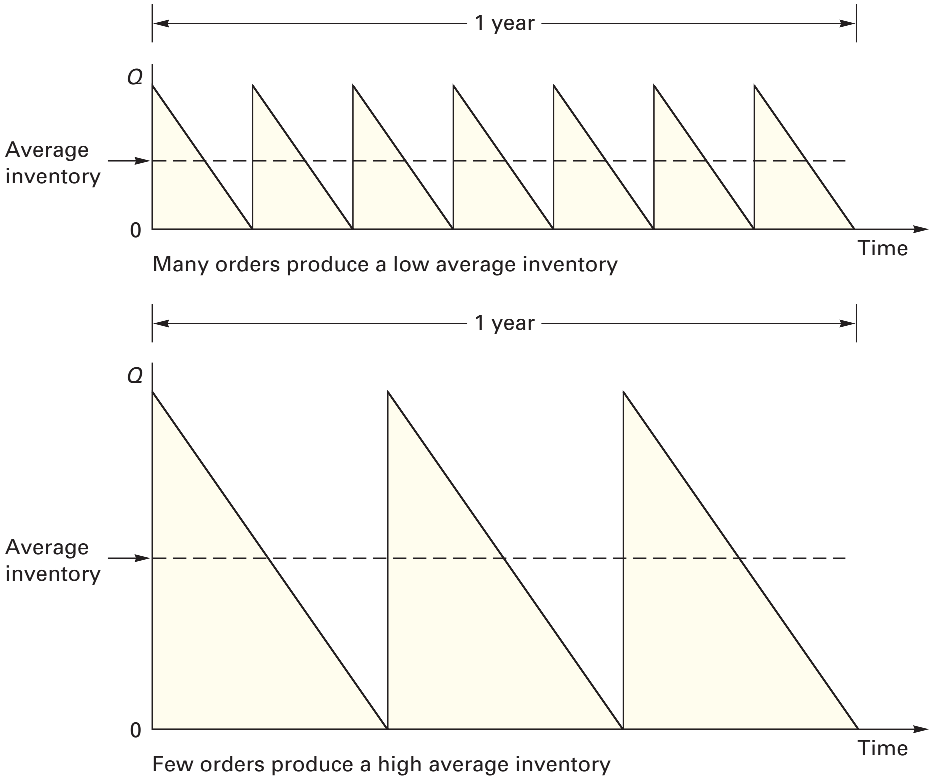
The basic model involves a number of assumptions. They are listed in Table 12.1.

Inventory ordering and usage occur in cycles. Figure 12.2 illustrates several inventory cycles. A cycle begins with receipt of an order of Q units, which are withdrawn at a constant rate over time. When the quantity on hand is just sufficient to satisfy demand during lead time, an order for Q units is submitted to the supplier. Because it is assumed that both the usage rate and the lead time do not vary, the order will be received at the precise instant that the inventory on hand falls to zero. Thus, orders are timed to avoid both excess stock and stockouts (i.e., running out of stock).

The optimal order quantity reflects a balance between carrying costs and ordering costs: As order size varies, one type of cost will increase while the other decreases. For example, if the order size is relatively small, the average inventory will be low, resulting in low carrying costs. However, a small order size will necessitate frequent orders, which will drive up annual ordering costs. Conversely, ordering large quantities at infrequent intervals can hold down annual ordering costs, but that would result in higher average inventory levels and therefore increased carrying costs. Figure 12.3 illustrates these two extremes.

Thus, the ideal solution is an order size that causes neither a few very large orders nor many small orders, but one that lies somewhere between. The exact amount to order will depend on the relative magnitudes of carrying and ordering costs.

FIGURE 12.3
Average inventory level and number of orders per year are inversely related: As one increases, the other decreases



Annual carrying cost is computed by multiplying the average amount of inventory on hand by the cost to carry one unit for one year, even though any given unit would not necessarily be held for a year. The average inventory is simply half of the order quantity: The amount on hand decreases steadily from Q units to 0, for an average of $(Q + 0)/2$, or $Q/2$. Using the symbol H to represent the average annual carrying cost per unit, the *total annual carrying cost* is

$$\text{Annual carrying cost} = \frac{Q}{2}H$$

where

Q = Order quantity in units
H = Holding (carrying) cost per unit

Carrying cost is thus a linear function of Q: Carrying costs increase or decrease in direct proportion to changes in the order quantity Q, as Figure 12.4A illustrates.

On the other hand, annual ordering cost will decrease as order size increases because, for a given annual demand, the larger the order size, the fewer the number of orders needed. For instance, if annual demand is 12,000 units and the order size is 1,000 units per order, there must be 12 orders over the year. But if $Q = 2{,}000$ units, only six orders will be needed; if $Q = 3{,}000$ units, only four orders will be needed. In general, the number of orders per year will be D/Q, where D = Annual demand and Q = Order size. Unlike carrying costs, ordering costs are relatively insensitive to order size; regardless of the amount of an order, certain activities must be done, such as determining how much is needed, periodically evaluating sources of supply, and preparing the invoice. Even inspection of the shipment to verify quality and quantity characteristics is not strongly influenced by order size since large shipments are sampled rather than completely inspected. Hence, ordering cost is treated as a constant. *Annual ordering cost* is a function of the number of orders per year and the ordering cost per order:

$$\text{Annual ordering cost} = \frac{D}{Q}S$$

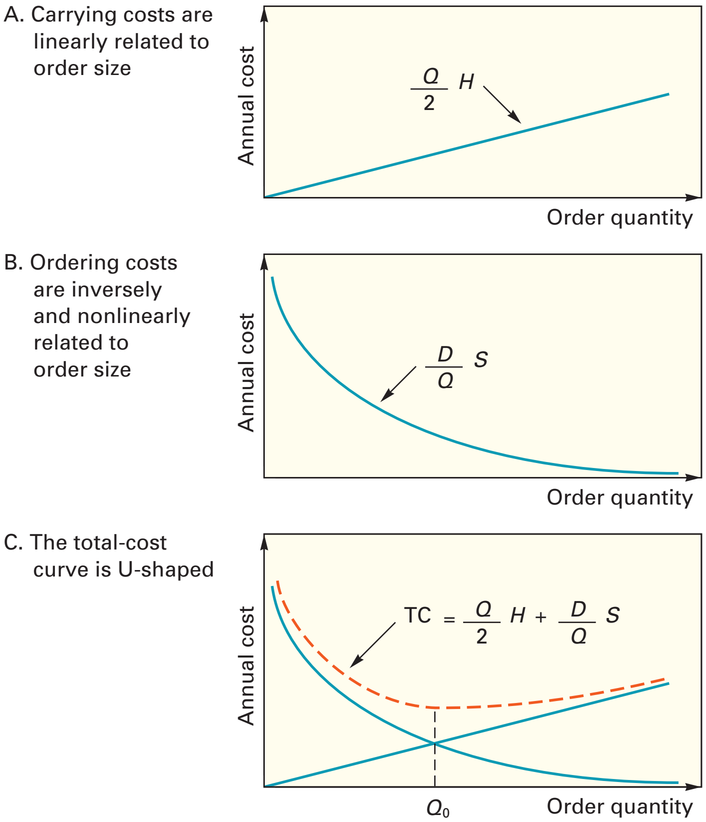


FIGURE 12.4
Carrying cost, ordering cost, and total cost curve

where

D = Demand, usually in units per year

S = Ordering cost

Because the number of orders per year, D/Q, decreases as Q increases, annual ordering cost is inversely related to order size, as Figure 12.4B illustrates.

The total annual cost (TC) associated with carrying and ordering inventory when Q units are ordered each time is

$$\text{TC} = \begin{matrix}\text{Annual}\\ \text{carrying}\\ \text{cost}\end{matrix} + \begin{matrix}\text{Annual}\\ \text{ordering}\\ \text{cost}\end{matrix} = \frac{Q}{2}H + \frac{D}{Q}S \tag{12–1}$$

(Note that D and H must be in the same units, e.g., months, years.) Figure 12.4C reveals that the total-cost curve is U-shaped (i.e., convex, with one minimum) and that *it reaches its minimum at the quantity where carrying and ordering costs are equal.* An expression for the optimal order quantity, Q_0, can be obtained using calculus.[1] The result is the formula

$$Q_0 = \sqrt{\frac{2DS}{H}} \tag{12–2}$$

[1]We can find the minimum point of the total-cost curve by differentiating TC with respect to Q, setting the result equal to zero, and solving for Q. Thus,

1. $\frac{d\text{TC}}{dQ} = \frac{dQ}{2}H + d(D/Q)S = H/2 - DS/Q^2$
2. $0 = H/2 - DS/Q^2$, so $Q^2 = \frac{2DS}{H}$ and $Q = \sqrt{\frac{2DS}{H}}$

Note that the second derivative is positive, which indicates a minimum has been obtained.

Thus, given annual demand, the ordering cost per order, and the annual carrying cost per unit, one can compute the optimal (economic) order quantity. The minimum total cost is then found by substituting Q_0 for Q in Formula 12–1.

The length of an order cycle (i.e., the time between orders) is

$$\text{Length of order cycle} = \frac{Q}{D} \qquad (12\text{–}3)$$

EXAMPLE 2

A local distributor for a national tire company expects to sell approximately 9,600 steel-belted radial tires of a certain size and tread design next year. Annual carrying cost is $16 per tire, and ordering cost is $75. The distributor operates 288 days a year.

a. What is the EOQ?
b. How many times per year does the store reorder?
c. What is the length of an order cycle?
d. What is the total annual cost if the EOQ quantity is ordered?

SOLUTION

D = 9,600 tires per year
H = $16 per unit per year
S = $75

a. $Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(9{,}600)75}{16}} = 300$ tires

b. Number of orders per year: $D/Q = \frac{9{,}600 \text{ tires}}{300 \text{ tires}} = 32.$

c. Length of order cycle: $Q/D = \frac{300 \text{ tires}}{9{,}600 \text{ tires/yr}} = 1/32$ of a year, which is $1/32 \times 288$, or nine workdays.

d. $\text{TC} = \text{Carrying cost} + \text{Ordering cost}$
$= (Q/2)H + (D/Q)S$
$= (300/2)16 + (9{,}600/300)75$
$= \$2{,}400 + \$2{,}400$
$= \$4{,}800$

Note that the ordering and carrying costs are equal at the EOQ, as illustrated in Figure 12.4C.

Carrying cost is sometimes stated as a percentage of the purchase price of an item rather than as a dollar amount per unit. However, as long as the percentage is converted into a dollar amount, the EOQ formula is still appropriate.

EXAMPLE 3

Piddling Manufacturing assembles security monitors. It purchases 3,600 black-and-white cathode ray tubes a year at $65 each. Ordering costs are $31, and annual carrying costs are 20 percent of the purchase price. Compute the optimal quantity and the total annual cost of ordering and carrying the inventory.

SOLUTION

D = 3,600 cathode ray tubes per year
S = $31
H = .20($65) = $13

$$Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(3{,}600)(31)}{13}} \approx 131 \text{ cathode ray tubes}$$

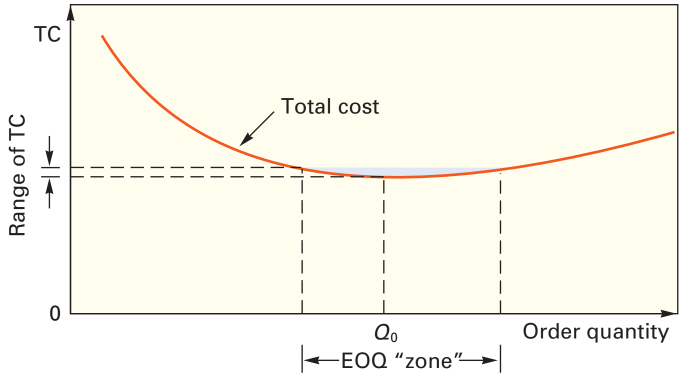


FIGURE 12.5
The total cost curve is relatively flat near the EOQ

$$\begin{aligned} TC &= \text{Carrying costs} + \text{Ordering costs} \\ &= (Q_0/2)H + (D/Q_0)S \\ &= (131/2)13 + (3{,}600/131)31 \\ &= \$852 + \$852 = \$1{,}704 \end{aligned}$$

Comment Holding and ordering costs, and annual demand, are typically estimated values rather than values that can be precisely determined, say, from accounting records. Holding costs are sometimes *designated* by management rather than computed. Consequently, the EOQ should be regarded as an *approximate* quantity rather than an exact quantity. Thus, rounding the calculated value is perfectly acceptable; stating a value to several decimal places would tend to give an unrealistic impression of the precision involved. An obvious question is: How good is this "approximate" EOQ in terms of minimizing cost? The answer is that the EOQ is fairly robust; the total cost curve is relatively flat near the EOQ, especially to the right of the EOQ. In other words, even if the resulting EOQ differs from the actual EOQ, total costs will not increase much at all. This is particularly true for quantities larger than the real EOQ, because the total cost curve rises very slowly to the right of the EOQ. (See Figure 12.5.)

Because the total cost curve is relatively flat around the EOQ, there can be some flexibility to modify the order quantity a bit from the EOQ (say, to achieve a round lot or full truckload) without incurring much of an increase in total cost.

Economic Production Quantity (EPQ)

The batch mode of production is widely used in production. Even in assembly operations, portions of the work are done in batches. The reason for this is that in certain instances, the capacity to produce a part exceeds the part's usage or demand rate. As long as production continues, inventory will continue to grow. In such instances, it makes sense to periodically produce such items in batches, or *lots,* instead of producing continually.

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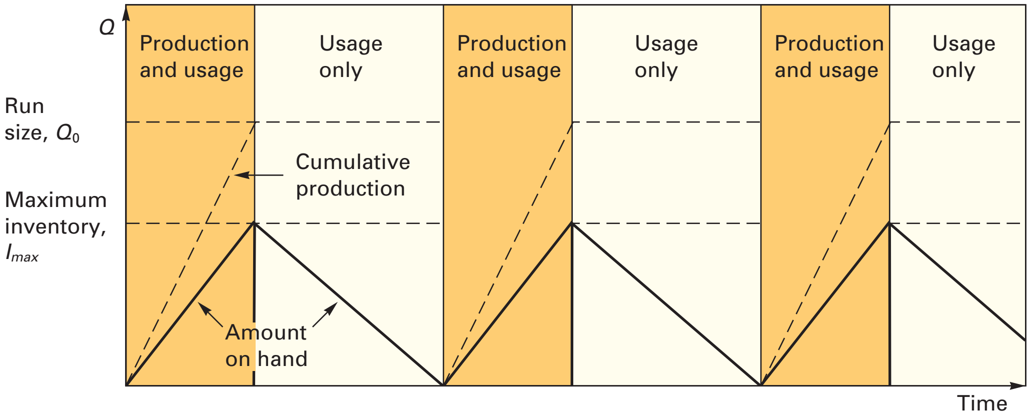
The assumptions of the EPQ model are similar to those of the EOQ model, except that instead of orders received in a single delivery, units are received incrementally during production. The assumptions are

1. Only one item is involved.
2. Annual demand is known.
3. The usage rate is constant.
4. Usage occurs continually, but production occurs periodically.

FIGURE 12.6 EOQ with incremental inventory replenishment



5. The production rate is constant.
6. Lead time does not vary.
7. There are no quantity discounts.

Figure 12.6 illustrates how inventory is affected by periodically producing a batch of a particular item.

During the production phase of the cycle, inventory builds up at a rate equal to the difference between production and usage rates. For example, if the daily production rate is 20 units and the daily usage rate is 5 units, inventory will build up at the rate of 20 − 5 = 15 units per day. As long as production occurs, the inventory level will continue to build; when production ceases, the inventory level will begin to decrease. Hence, the inventory level will be maximum at the point where production ceases. When the amount of inventory on hand is exhausted, production is resumed, and the cycle repeats itself.

Because the company makes the product itself, there are no ordering costs as such. Nonetheless, with every production run (batch) there are setup costs—the costs required to prepare the equipment for the job, such as cleaning, adjusting, and changing tools and fixtures. Setup costs are analogous to ordering costs because they are independent of the lot (run) size. They are treated in the formula in exactly the same way. The larger the run size, the fewer the number of runs needed and, hence, the lower the annual setup cost. The number of runs or batches per year is D/Q, and the annual setup cost is equal to the number of runs per year times the setup cost, S, per run: $(D/Q)S$.

The total cost is

$$TC_{min} = \text{Carrying cost} + \text{Setup cost} = \left(\frac{I_{max}}{2}\right)H + (D/Q_0)S \tag{12–4}$$

where

I_{max} = Maximum inventory

The economic run quantity is

$$Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} \tag{12–5}$$

where

p = Production or delivery rate
u = Usage rate

The cycle time (the time between orders or between the beginnings of runs) for the economic run size model is a function of the run size and usage (demand) rate:

$$\text{Cycle time} = \frac{Q_0}{u} \qquad (12\text{–}6)$$

Similarly, the run time (the production phase of the cycle) is a function of the run (lot) size and the production rate:

$$\text{Run time} = \frac{Q_0}{p} \qquad (12\text{–}7)$$

The maximum and average inventory levels are

$$I_{max} = \frac{Q_0}{p}(p - u) \quad \text{and} \quad I_{average} = \frac{I_{max}}{2} \qquad (12\text{–}8)$$

EXAMPLE 4

A toy manufacturer uses 48,000 rubber wheels per year for its popular dump truck series. The firm makes its own wheels, which it can produce at a rate of 800 per day. The toy trucks are assembled uniformly over the entire year. Carrying cost is $1 per wheel a year. Setup cost for a production run of wheels is $45. The firm operates 240 days per year. Determine the

a. Optimal run size.
b. Minimum total annual cost for carrying and setup.
c. Cycle time for the optimal run size.
d. Run time.

SOLUTION

D = 48,000 wheels per year
S = $45
H = $1 per wheel per year
p = 800 wheels per day
u = 48,000 wheels per 240 days, or 200 wheels per day

a. $$Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(48{,}000)45}{1}}\sqrt{\frac{800}{800-200}} = 2{,}400 \text{ wheels}$$

b. $$\text{TC}_{min} = \text{Carrying cost} + \text{Setup cost} = \left(\frac{I_{max}}{2}\right)H + (D/Q_0)S$$

Thus, you must first compute I_{max}:

$$I_{max} = \frac{Q_0}{p}(p - u) = \frac{2{,}400}{800}(800 - 200) = 1{,}800 \text{ wheels}$$

$$\text{TC} = \frac{1{,}800}{2} \times \$1 + \frac{48{,}000}{2{,}400} \times \$45 = \$900 + \$900 = \$1{,}800$$

Note again the equality of cost (in this example, setup and carrying costs) at the EOQ.

c. $$\text{Cycle time} = \frac{Q_0}{u} = \frac{2{,}400 \text{ wheels}}{200 \text{ wheels per day}} = 12 \text{ days}$$

Thus, a run of wheels will be made every 12 days.

d. $$\text{Run time} = \frac{Q_0}{p} = \frac{2{,}400 \text{ wheels}}{800 \text{ wheels per day}} = 3 \text{ days}$$

Thus, each run will require three days to complete.

TABLE 12.2
Price list for extra-wide gauze strips

Order Quantity	Price per Box
1 to 44	$2.00
45 to 69	1.70
70 or more	1.40

Quantity Discounts

Quantity discounts Price reductions for large orders.

Quantity discounts are price reductions for large orders offered to customers to induce them to buy in large quantities. For example, a Chicago surgical supply company publishes the price list shown in Table 12.2 for boxes of gauze strips. Note that the price per box decreases as order quantity increases.

If quantity discounts are offered, the buyer must weigh the potential benefits of reduced purchase price and fewer orders that will result from buying in large quantities against the increase in carrying costs caused by higher average inventories. The buyer's goal with quantity discounts is to select the order quantity that will minimize total cost, where total cost is the sum of carrying cost, ordering cost, *and* purchasing (i.e., product) cost:

$$\text{TC} = \text{Carrying cost} + \text{Ordering cost} + \text{Purchasing cost} \qquad (12\text{–}9)$$

$$= \left(\frac{Q}{2}\right)H + \left(\frac{D}{Q}\right)S + PD$$

where

P = Unit price

Recall that in the basic EOQ model, determination of order size does not involve the purchasing cost. The rationale for not including unit price is that under the assumption of no quantity discounts, price per unit is the same for all order sizes. Inclusion of unit price in the total-cost computation in that case would merely increase the total cost by the amount P times D. A graph of total annual purchase cost versus quantity would be a horizontal line. Hence, including purchasing costs would merely raise the total-cost curve by the same amount (PD) at every point. That would not change the EOQ. (See Figure 12.7.)

When quantity discounts are offered, there is a separate U-shaped total-cost curve for each unit price. Again, including unit prices merely raises each curve by a constant amount. However, because the unit prices are all different, each curve is raised by a different amount: Smaller unit prices will raise a total-cost curve less than larger unit prices. Note that no one curve applies to the entire range of quantities; each curve applies to only a *portion* of the range. (See Figure 12.8.) Hence, the applicable or *feasible* total cost is initially on the curve with the highest unit price and then drops down, curve by curve, at the *price breaks,* which are the minimum quantities needed to obtain the discounts. Thus, in Table 12.2, the price breaks for gauze strips are at 45 and 70 boxes. The result is a total-cost curve with *steps* at the price breaks.

Even though each curve has a minimum, those points are not necessarily feasible. For example, the minimum point for the $1.40 curve in Figure 12.8 appears to be about 65 units. However, the price list shown in Table 12.2 indicates that an order size of 65 boxes will involve a unit price of $1.70. The actual total-cost curve is denoted by the solid lines; only those price–quantity combinations are feasible. The objective of the quantity discount model is to identify the order quantity that will represent the lowest total cost for the entire set of curves.

There are two general cases of the model. In one, carrying costs are constant (e.g., $2 per unit); in the other, carrying costs are stated as a percentage of purchase price (e.g., 20 percent of unit price). When carrying costs are constant, there will be a single minimum point.

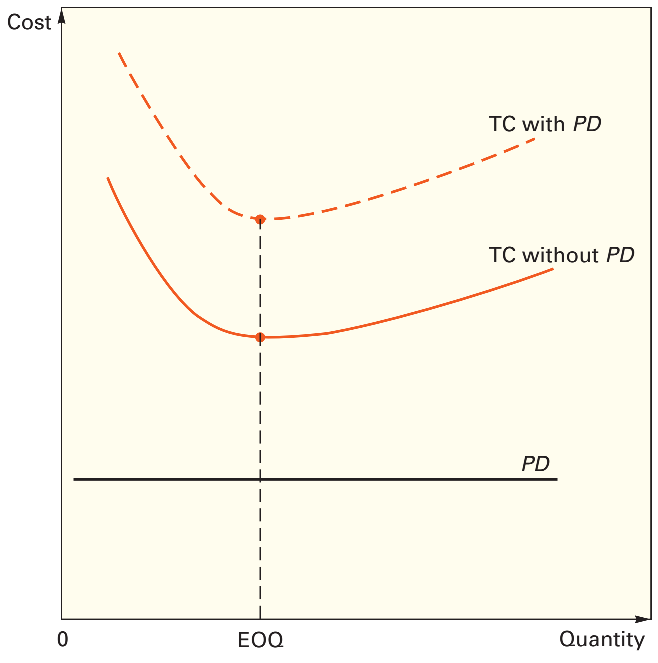


FIGURE 12.7
Adding *PD* doesn't change the EOQ

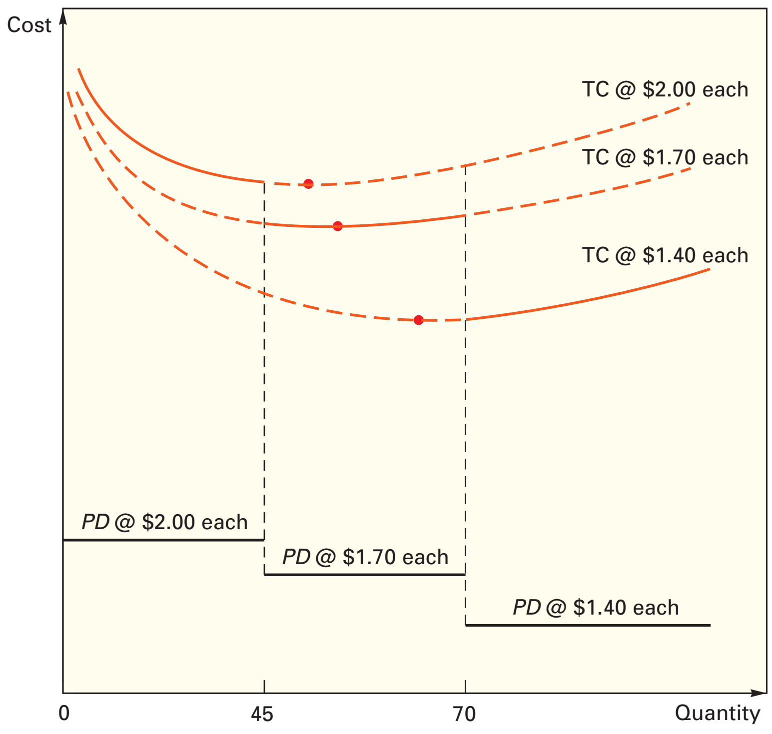


FIGURE 12.8
The total-cost curve with quantity discounts is composed of a portion of the total-cost curve for each price

All curves will have their minimum point at the same quantity. Consequently, the total-cost curves line up vertically, differing only in that the lower unit prices are reflected by lower total-cost curves as shown in Figure 12.9A. (For purposes of illustration, the horizontal purchasing cost lines have been omitted.)

When carrying costs are specified as a percentage of unit price, each curve will have a different minimum point. Because carrying costs are a percentage of price, lower prices

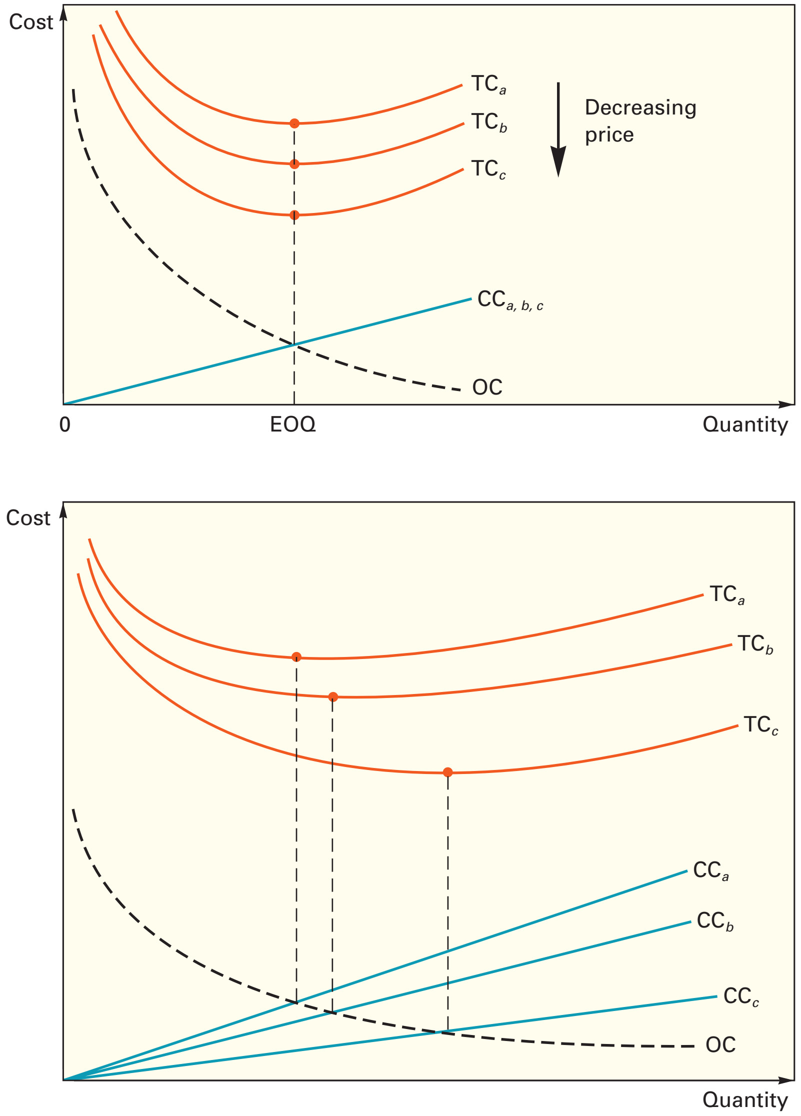


FIGURE 12.9
Comparison of TC curves for constant carrying costs and carrying costs that are a percentage of unit costs

A. When carrying costs are constant, all curves have their minimum points at the same quantity.

B. When carrying costs are stated as a percentage of unit price, the minimum points do not line up.

will mean lower carrying costs and larger minimum points. Thus, as price decreases, each curve's minimum point will be to the right of the next higher curve's minimum point. (See Figure 12.9B.)

The procedure for determining the overall EOQ differs slightly, depending on which of these two cases is relevant. For carrying costs that are constant, the procedure is as follows:

1. Compute the common minimum point.
2. Only one of the unit prices will have the minimum point in its feasible range since the ranges do not overlap. Identify that range.
   a. If the feasible minimum point is on the lowest price range, that is the optimal order quantity.
   b. If the feasible minimum point is in any other range, compute the total cost for the minimum point and for the price breaks of all *lower* unit costs. Compare the total costs; the quantity (minimum point or price break) that yields the lowest total cost is the optimal order quantity.

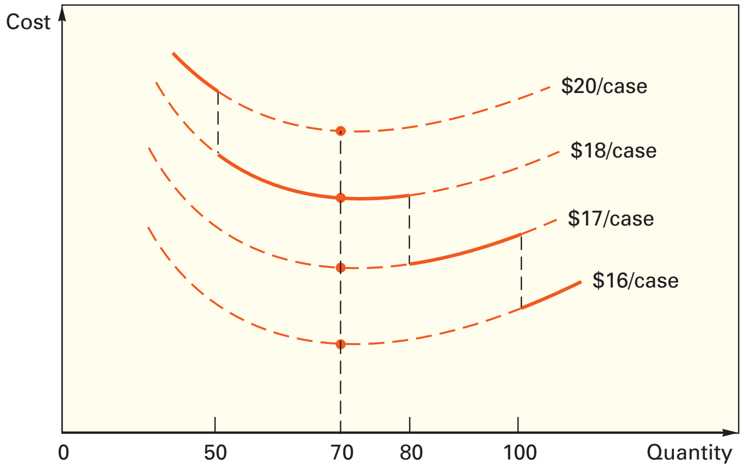


FIGURE 12.10
Total-cost curves for Example 5

EXAMPLE 5

The maintenance department of a large hospital uses about 816 cases of liquid cleanser annually. Ordering costs are \$12, carrying costs are \$4 per case a year, and the new price schedule indicates that orders of less than 50 cases will cost \$20 per case, 50 to 79 cases will cost \$18 per case, 80 to 99 cases will cost \$17 per case, and larger orders will cost \$16 per case. Determine the optimal order quantity and the total cost.

SOLUTION

See Figure 12.10:

D = 816 cases per year S = \$12 H = \$4 per case per year

Range	Price
1 to 49	\$20
50 to 79	18
80 to 99	17
100 or more	16

1. Compute the common EOQ: $= \sqrt{\dfrac{2DS}{H}} = \sqrt{\dfrac{2(816)12}{4}} = 69.97 \approx 70$ cases

2. The 70 cases can be bought at \$18 per case because 70 falls in the range of 50 to 79 cases. The total cost to purchase 816 cases a year, at the rate of 70 cases per order, will be

$$\begin{aligned} TC_{70} &= \text{Carrying cost} + \text{Order cost} + \text{Purchase cost} \\ &= (Q/2)H + (D/Q_0)S + PD \\ &= (70/2)4 + (816/70)12 + 18(816) = \$14{,}968 \end{aligned}$$

Because lower cost ranges exist, each must be checked against the minimum cost generated by 70 cases at \$18 each. In order to buy at \$17 per case, at least 80 cases must be purchased. (Because the TC curve is rising, 80 cases will have the lowest TC for that curve's feasible region.) The total cost at 80 cases will be

$$TC_{80} = (80/2)4 + (816/80)12 + 17(816) = \$14{,}154$$

To obtain a cost of \$16 per case, at least 100 cases per order are required, and the total cost at that price break will be

$$TC_{100} = (100/2)4 + (816/100)12 + 16(816) = \$13{,}354$$

Therefore, because 100 cases per order yields the lowest total cost, 100 cases is the overall optimal order quantity.

When carrying costs are expressed as a percentage of price, determine the best purchase quantity with the following procedure:

1. Beginning with the lowest unit price, compute the minimum points for each price range until you find a feasible minimum point (i.e., until a minimum point falls in the quantity range for its price).
2. If the minimum point for the lowest unit price is feasible, it is the optimal order quantity. If the minimum point is not feasible in the lowest price range, compare the total cost at the price break for all *lower* prices with the total cost of the feasible minimum point. The quantity that yields the lowest total cost is the optimum.

EXAMPLE 6

Surge Electric uses 4,000 toggle switches a year. Switches are priced as follows: 1 to 499, 90 cents each; 500 to 999, 85 cents each; and 1,000 or more, 80 cents each. It costs approximately \$30 to prepare an order and receive it, and carrying costs are 40 percent of purchase price per unit on an annual basis. Determine the optimal order quantity and the total annual cost.

SOLUTION

See Figure 12.11:

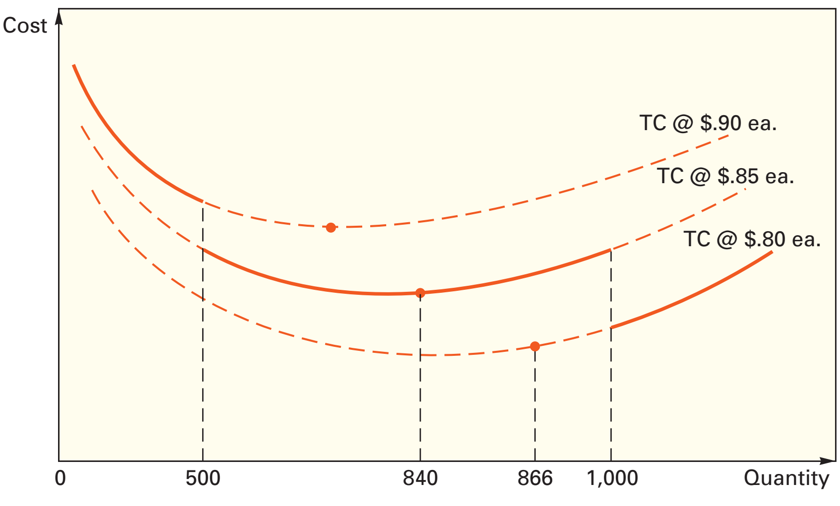
$D = 4{,}000$ switches per year $\quad S = \$30 \quad H = .40P$

Range	Unit Price	*H*
1 to 499	\$0.90	.40(0.90) = .36
500 to 999	\$0.85	.40(0.85) = .34
1,000 or more	\$0.80	.40(0.80) = .32

Find the minimum point for each price, starting with the lowest price, until you locate a feasible minimum point.

$$\text{Minimum point}_{0.80} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(4{,}000)30}{.32}} = 866 \text{ switches}$$

FIGURE 12.11
Total-cost curves for Example 6



Because an order size of 866 switches will cost \$0.85 each rather than \$0.80 each, 866 is not a feasible minimum point for \$0.80 per switch. Next, try \$0.85 per unit.

$$\text{Minimum point}_{0.85} = \sqrt{\frac{2(4{,}000)30}{.34}} = 840 \text{ switches}$$

This is feasible; it falls in the \$0.85 per switch range of 500 to 999.

Now compute the total cost for 840, and compare it to the total cost of the minimum quantity necessary to obtain a price of \$0.80 per switch.

$$\begin{aligned}
\text{TC} &= \text{Carrying costs} + \text{Ordering costs} + \text{Purchasing costs} \\
&= \left(\frac{Q}{2}\right)H + \left(\frac{D}{Q}\right)S + PD \\
\text{TC}_{840} &= \frac{840}{2}(.34) + \frac{4{,}000}{840}(30) + 0.85(4{,}000) = \$3{,}686 \\
\text{TC}_{1{,}000} &= \frac{1{,}000}{2}(.32) + \frac{4{,}000}{1{,}000}(30) + 0.80(4{,}000) = \$3{,}480
\end{aligned}$$

Thus, the minimum-cost order size is 1,000 switches.

WHEN TO REORDER WITH EOQ ORDERING

EOQ models answer the question of how much to order, but not the question of when to order. The latter is the function of models that identify the **reorder point (ROP)** in terms of a *quantity:* The reorder point occurs when the quantity on hand drops to a predetermined amount. That amount generally includes expected demand during lead time and perhaps an extra cushion of stock, which serves to reduce the probability of experiencing a stockout during lead time. Note that in order to know when the reorder point has been reached, a *perpetual* inventory is required.

Reorder point (ROP) When the quantity on hand of an item drops to this amount, the item is reordered.

The goal in ordering is to place an order when the amount of inventory on hand is sufficient to satisfy demand during the time it takes to receive that order (i.e., lead time). There are four determinants of the reorder point quantity:

1. The rate of demand (usually based on a forecast).
2. The lead time.
3. The extent of demand and/or lead time variability.
4. The degree of stockout risk acceptable to management.

If demand and lead time are both constant, the reorder point is simply

$$\text{ROP} = d \times \text{LT} \quad (12\text{–}10)$$

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where

d = Demand rate (units per day or week)

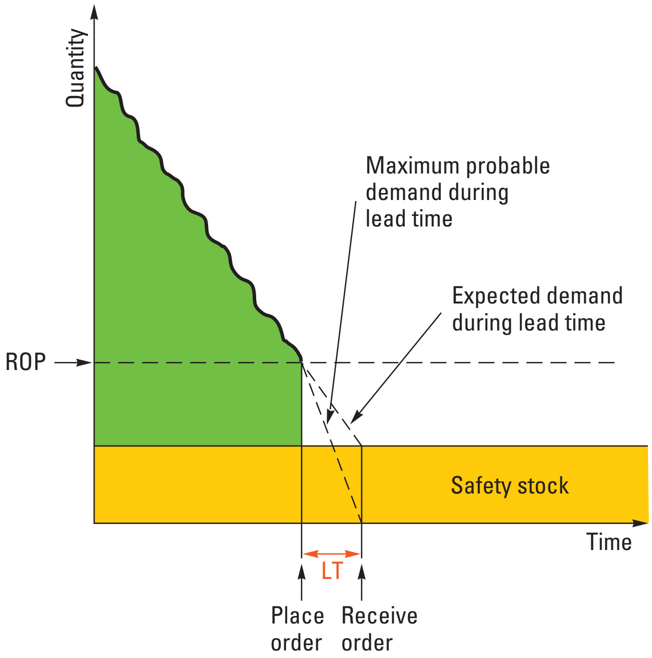
LT = Lead time in days or weeks

Note: Demand and lead time must be expressed in the same time units.

EXAMPLE 7

Tingly takes Two-a-Day vitamins, which are delivered to his home by a routeman seven days after an order is called in. At what point should Tingly reorder?

FIGURE 12.12
Safety stock reduces risk of stockout during lead time



SOLUTION

$$\text{Usage} = 2 \text{ vitamins a day}$$
$$\text{Lead time} = 7 \text{ days}$$
$$\text{ROP} = \text{Usage} \times \text{Lead time}$$
$$= 2 \text{ vitamins per day} \times 7 \text{ days} = 14 \text{ vitamins}$$

Thus, Tingly should reorder when 14 vitamin tablets are left.

Safety stock Stock that is held in excess of expected demand due to variable demand and/or lead time.

When variability is present in demand or lead time, it creates the possibility that actual demand will exceed expected demand. Consequently, it becomes necessary to carry additional inventory, called **safety stock,** to reduce the risk of running out of inventory (a stockout) during lead time. The reorder point then increases by the amount of the safety stock:

$$\text{ROP} = \begin{matrix}\text{Expected demand} \\ \text{during lead time}\end{matrix} + \text{Safety stock} \quad (12\text{–}11)$$

For example, if expected demand during lead time is 100 units, and the desired amount of safety stock is 10 units, the ROP would be 110 units.

Figure 12.12 illustrates how safety stock can reduce the risk of a stockout during lead time (LT). Note that stockout protection is needed only during lead time. If there is a sudden surge at any point during the cycle, that will trigger another order. Once that order is received, the danger of an imminent stockout is negligible.

Service level Probability that demand will not exceed supply during lead time.

Because it costs money to hold safety stock, a manager must carefully weigh the cost of carrying safety stock against the reduction in stockout risk it provides. The customer *service level* increases as the risk of stockout decreases. Order cycle **service level** can be defined as the probability that demand will not exceed supply during lead time (i.e., that the amount of stock on hand will be sufficient to meet demand). Hence, a service level of 95 percent implies a probability of 95 percent that demand will not exceed supply during lead time. An equivalent statement that demand will be satisfied in 95 percent of such instances does *not* mean that 95 percent of demand will be satisfied. The risk of a stockout is the complement of service level; a customer service level of 95 percent implies a stockout risk of 5 percent. That is,

$$\text{Service level} = 100 \text{ percent} - \text{Stockout risk}$$

Later you will see how the order cycle service level relates to the *annual* service level.

Consider for a moment the importance of stockouts. When a stockout occurs, demand cannot be satisfied at that time. In manufacturing operations, stockouts mean that jobs will be delayed and additional costs will be incurred. If the stockout involves parts for an assembly line, or spare parts for a machine or conveyor belt on the line, the line will have to shut down, typically at a very high cost per hour, until parts can be obtained. For service operations, stockouts mean that services cannot be completed on time. Aside from the added cost that results from the time delay, there is not only the matter of customer dissatisfaction but also the fact that schedules will be disrupted, sometimes creating a "domino effect" on following jobs. In the retail sector, stockouts create a competitive *disadvantage* that can result in customer dissatisfaction and, ultimately, the loss of customers.

The amount of safety stock that is appropriate for a given situation depends on the following factors:

1. The average demand rate and average lead time.
2. Demand and lead time variability.
3. The desired service level.

For a given order cycle service level, the greater the variability in either demand rate or lead time, the greater the amount of safety stock that will be needed to achieve that service level. Similarly, for a given amount of variation in demand rate or lead time, achieving an increase in the service level will require increasing the amount of safety stock. Selection of a service level may reflect stockout costs (e.g., lost sales, customer dissatisfaction) or it might simply be a policy variable (e.g., the manager wants to achieve a specified service level for a certain item).

Let us look at several models that can be used in cases when variability is present. The first model can be used if an estimate of expected demand during lead time and its standard deviation are available. The formula is

$$\text{ROP} = \begin{matrix}\text{Expected demand} \\ \text{during lead time}\end{matrix} + z\sigma_{d\text{LT}} \qquad (12\text{–}12)$$

where

z = Number of standard deviations

$\sigma_{d\text{LT}}$ = The standard deviation of lead time demand

The models generally assume that any variability in demand rate or lead time can be adequately described by a normal distribution. However, this is not a strict requirement; the models provide approximate reorder points even where actual distributions depart from normal.

The value of z (see Figure 12.13) used in a particular instance depends on the stockout risk that the manager is willing to accept. Generally, the smaller the risk the manager is willing to accept, the greater the value of z. Use Appendix B, Table B, to obtain the value of z, given a desired service level for lead time.

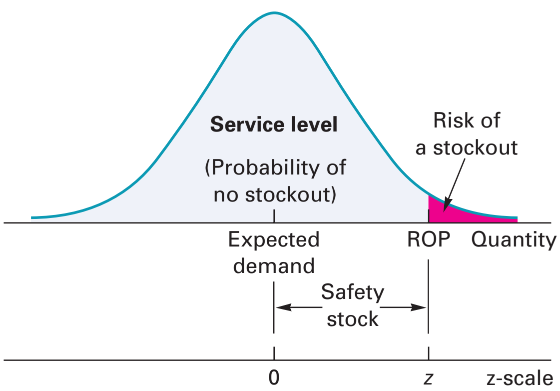


FIGURE 12.13
The ROP based on a normal distribution of lead time demand

EXAMPLE 8

Suppose that the manager of a construction supply house determined from historical records that demand for sand during lead time averages 50 tons. In addition, suppose the manager determined that demand during lead time could be described by a normal distribution that has a mean of 50 tons and a standard deviation of 5 tons. Answer these questions, assuming that the manager is willing to accept a stockout risk of no more than 3 percent:

a. What value of z is appropriate?

b. How much safety stock should be held?

c. What reorder point should be used?

SOLUTION

Expected lead time demand = 50 tons

σ_{dLT} = 5 tons

Risk = 3 percent

a. From Appendix B, Table B, using a service level of $1 - .03 = .9700$, you obtain a value of $z = +1.88$.

b. Safety stock $= z\sigma_{dLT} = 1.88(5) = 9.40$ tons

c. ROP = Expected lead time demand + Safety stock = 50 + 9.40 = 59.40 tons

When data on lead time demand are not readily available, Formula 12–12 cannot be used. Nevertheless, data are generally available on daily or weekly demand, and on the length of lead time. Using those data, a manager can determine whether demand and/or lead time is variable, if variability exists in one or both, and the related standard deviation(s). For those situations, one of the following formulas can be used:

If only demand is variable, then $\sigma_{dLT} = \sigma_d\sqrt{\text{LT}}$, and the reorder point is

$$\text{ROP} = \bar{d} \times \text{LT} + z\sigma_d\sqrt{\text{LT}} \tag{12–13}$$

where

$\bar{d}$ = *Average* daily or weekly demand

σ_d = Standard deviation of demand per day or week

LT = Lead time in days or weeks

If only lead time is variable, then $\sigma_{dLT} = d\sigma_{\text{LT}}$, and the reorder point is

$$\text{ROP} = d \times \overline{\text{LT}} + zd\sigma_{\text{LT}} \tag{12–14}$$

where

d = Daily or weekly demand

$\overline{\text{LT}}$ = *Average* lead time in days or weeks

σ_{LT} = Standard deviation of lead time in days or weeks

If both demand and lead time are variable, then

$$\sigma_{dLT} = \sqrt{\overline{\text{LT}}\sigma_d^2 + \bar{d}^2\sigma_{\text{LT}}^2}$$

and the reorder point is

$$\text{ROP} = \bar{d} \times \overline{\text{LT}} + z\sqrt{\overline{\text{LT}}\sigma_d^2 + \bar{d}^2\sigma_{\text{LT}}^2} \tag{12–15}$$

Note: Each of these models assumes that demand and lead time are *independent.*

EXAMPLE 9

A restaurant uses an average of 50 jars of a special sauce each week. Weekly usage of sauce has a standard deviation of 3 jars. The manager is willing to accept no more than a 10 percent risk of stockout during lead time, which is two weeks. Assume the distribution of usage is normal.

a. Which of the above formulas is appropriate for this situation? Why?
b. Determine the value of z.
c. Determine the ROP.

SOLUTION

$\bar{d}$ = 50 jars per week LT = 2 weeks
σ_d = 3 jars per week Acceptable risk = 10 percent, so service level is .90

a. Because only demand is variable (i.e., has a standard deviation), Formula 12–13 is appropriate.
b. From Appendix B, Table B, using a service level of .9000, you obtain $z = +1.28$.
c. $\text{ROP} = \bar{d} \times \text{LT} + z\sigma_d\sqrt{\text{LT}} = 50 \times 2 + 1.28(3)\sqrt{2} = 100 + 5.43 = 105.43$.
Because the inventory is discrete units (jars), we round this amount to 106. (Generally, round up.)

Note that a 2-bin ordering system (see p. 545) involves ROP reordering: The quantity in the second bin is equal to the ROP.

Comment The logic of the three formulas for the reorder point may not be immediately obvious. The first part of each formula is the expected demand, which is the product of daily (or weekly) demand and the number of days (or weeks) of lead time. The second part of the formula is z times the standard deviation of lead time demand. For the formula in which only demand is variable, daily (or weekly) demand is assumed to be normally distributed and has the same mean and standard deviation (see Figure 12.14). The standard deviation of demand for the entire lead time is found by summing the *variances* of daily (or weekly) demands, and then finding the square root of that number because, unlike variances, standard deviations are not additive. Hence, if the daily standard deviation is σ_d, the *variance* is σ_d^2, and if lead time is four days, the variance of lead time demand will equal the sum of the 4 variances, which is $4\sigma_d^2$. The standard deviation of lead time demand will be the square root of this, which is equal to $2\sigma_d$. In general, this becomes $\sqrt{\text{LT}}\sigma_d$ and, hence, the last part of Formula 12–13.

When only lead time is variable, the explanation is much simpler. The standard deviation of lead time demand is equal to the constant daily demand multiplied by the standard deviation of lead time.

When both demand and lead time are variable, the formula appears truly impressive. However, it is merely the result of squaring the standard deviations of the two previous formulas to obtain their variances, summing them, and then taking the square root.

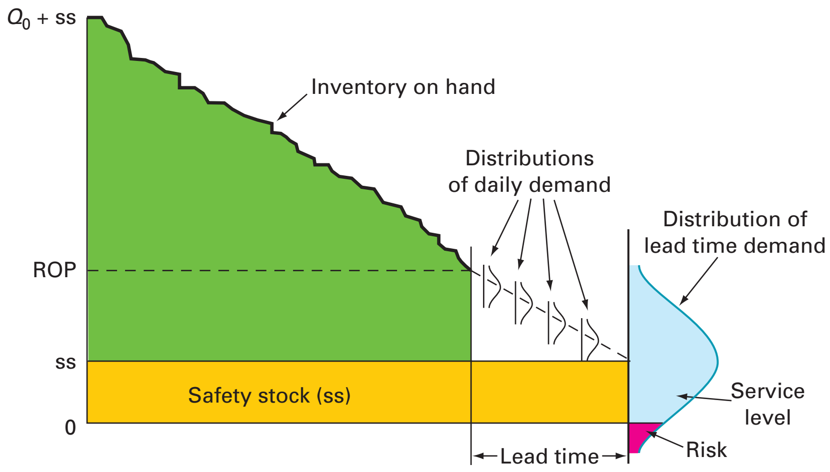


FIGURE 12.14
Lead time demand

Shortages and Service Levels

The ROP computation does not reveal the expected *amount* of shortage for a given lead time service level. The expected number of units short can, however, be very useful to a manager. This quantity can easily be determined from the same information used to compute the ROP, with one additional piece of information (see Table 12.3). Use of the table assumes that the distribution of lead time demand can be adequately represented by a normal distribution. If it can, the expected number of units short in each order cycle is given by this formula:

$$E(n) = E(z)\sigma_{dLT} \qquad (12\text{–}16)$$

where

$E(n)$ = Expected number of units short per order cycle

$E(z)$ = Standardized number of units short obtained from Table 12.3

σ_{dLT} = Standard deviation of lead time demand

EXAMPLE 10

Suppose the standard deviation of lead time demand is known to be 20 units. Lead time demand is approximately normal.

a. For a lead time service level of 90 percent, determine the expected number of units short for any order cycle.

b. What lead time service level would an expected shortage of 2 units imply?

SOLUTION

σ_{dLT} = 20 units

a. Lead time (cycle) service level = .90. From Table 12.3, $E(z) = 0.048$. Using Formula 11–16, $E(n) = 0.048(20 \text{ units}) = 0.96$, or about 1 unit.

b. For the case where $E(n) = 2$, you must solve for $E(z)$ and then use Table 12.3 to determine the lead time service that implies. Thus, $E(n) = E(z)\sigma_{dLT}$, so $E(z) = E(n)/\sigma_{dLT} = 2/20 = 0.100$. From Table 12.3, this implies a service level of approximately 81.7 percent (interpolating).

The expected number of units short is just that—an expected or *average* amount; the exact number of units short in any given cycle will be an amount close to that. Moreover, if discrete items are involved, the actual number of units short in any cycle will be an integer.

Having determined the expected number of units short for an order cycle, you can determine the expected number of units short per year. It is simply the expected number of units short per cycle multiplied by the number of cycles (orders) per year. Thus,

$$E(N) = E(n)\frac{D}{Q} \qquad (12\text{–}17)$$

where

$E(N)$ = Expected number of units short per year

EXAMPLE 11

Given the following information, determine the expected number of units short per year.

$D = 1{,}000 \qquad Q = 250 \qquad E(n) = 2.5$

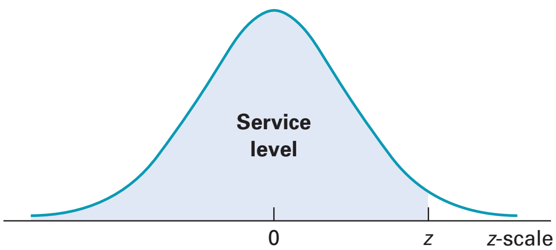
SOLUTION

Using the formula $E(N) = E(n)\frac{D}{Q}$,

$$E(N) = 2.5\left(\frac{1{,}000}{250}\right) = 10.0 \text{ units per year}$$

TABLE 12.3 Normal distribution service levels and unit normal loss function

z	Lead Time Service Level	$E(z)$	z	Lead Time Service Level	$E(z)$	z	Lead Time Service Level	$E(z)$	z	Lead Time Service Level	$E(z)$
−2.40	.0082	2.403	−.80	.2119	.920	0.80	.7881	.120	2.40	.9918	.0030
−2.36	.0091	2.363	−.76	.2236	.889	0.84	.7995	.112	2.44	.9927	.0020
−2.32	.0102	2.323	−.72	.2358	.858	0.88	.8106	.104	2.48	.9934	.0020
−2.28	.0113	2.284	−.68	.2483	.828	0.92	.8212	.097	2.52	.9941	.0020
−2.24	.0125	2.244	−.64	.2611	.798	0.96	.8315	.089	2.56	.9948	.0020
−2.20	.0139	2.205	−.60	.2743	.769	1.00	.8413	.083	2.60	.9953	.0010
−2.16	.0154	2.165	−.56	.2877	.740	1.04	.8508	.077	2.64	.9959	.0010
−2.12	.0170	2.126	−.52	.3015	.712	1.08	.8599	.071	2.68	.9963	.0010
−2.08	.0188	2.087	−.48	.3156	.684	1.12	.8686	.066	2.72	.9967	.0010
−2.04	.0207	2.048	−.44	.3300	.657	1.16	.8770	.061	2.76	.9971	.0010
−2.00	.0228	2.008	−.40	.3446	.630	1.20	.8849	.056	2.80	.9974	.0008
−1.96	.0250	1.969	−.36	.3594	.597	1.24	.8925	.052	2.84	.9977	.0007
−1.92	.0274	1.930	−.32	.3745	.576	1.28	.8997	.048	2.88	.9980	.0006
−1.88	.0301	1.892	−.28	.3897	.555	1.32	.9066	.044	2.92	.9982	.0005
−1.84	.0329	1.853	−.24	.4052	.530	1.36	.9131	.040	2.96	.9985	.0004
−1.80	.0359	1.814	−.20	.4207	.507	1.40	.9192	.037	3.00	.9987	.0004
−1.76	.0392	1.776	−.16	.4364	.484	1.44	.9251	.034	3.04	.9988	.0003
−1.72	.0427	1.737	−.12	.4522	.462	1.48	.9306	.031	3.08	.9990	.0003
−1.68	.0465	1.699	−.08	.4681	.440	1.52	.9357	.028	3.12	.9991	.0002
−1.64	.0505	1.661	−.04	.4840	.419	1.56	.9406	.026	3.16	.9992	.0002
−1.60	.0548	1.623	.00	.5000	.399	1.60	.9452	.023	3.20	.9993	.0002
−1.56	.0594	1.586	.04	.5160	.379	1.64	.9495	.021	3.24	.9994	.0001
−1.52	.0643	1.548	.08	.5319	.360	1.68	.9535	.019	3.28	.9995	.0001
−1.48	.0694	1.511	.12	.5478	.342	1.72	.9573	.017	3.32	.9995	.0001
−1.44	.0749	1.474	.16	.5636	.324	1.76	.9608	.016	3.36	.9996	.0001
−1.40	.0808	1.437	.20	.5793	.307	1.80	.9641	.014	3.40	.9997	.0001
−1.36	.0869	1.400	.24	.5948	.290	1.84	.9671	.013			
−1.32	.0934	1.364	.28	.6103	.275	1.88	.9699	.012			
−1.28	.1003	1.328	.32	.6255	.256	1.92	.9726	.010			
−1.24	.1075	1.292	.36	.6406	.237	1.96	.9750	.009			
−1.20	.1151	1.256	.40	.6554	.230	2.00	.9772	.008			
−1.16	.1230	1.221	.44	.6700	.217	2.04	.9793	.008			
−1.12	.1314	1.186	.48	.6844	.204	2.08	.9812	.007			
−1.08	.1401	1.151	.52	.6985	.192	2.12	.9830	.006			
−1.04	.1492	1.117	.56	.7123	.180	2.16	.9846	.005			
−1.00	.1587	1.083	.60	.7257	.169	2.20	.9861	.005			
−.96	.1685	1.049	.64	.7389	.158	2.24	.9875	.004			
−.92	.1788	1.017	.68	.7517	.148	2.28	.9887	.004			
−.88	.1894	0.984	.72	.7642	.138	2.32	.9898	.003			
−.84	.2005	0.952	.76	.7764	.129	2.36	.9909	.003			



It is sometimes convenient to think of service level in annual terms. One definition of annual service level is the percentage of demand filled directly from inventory. This is also known as the *fill rate*. Thus, if $D = 1{,}000$, and 990 units were filled directly from inventory (shortages totaling 10 units over the year were recorded), the annual service level (fill rate) would be $990/1{,}000 = 99$ percent. The annual service level and the lead time service level can be related using the following formula:

$$\text{SL}_{\text{annual}} = 1 - \frac{E(N)}{D} \qquad (12\text{–}18)$$

Using Formulas 12–17 and 12–16,

$$E(N) = E(n)D/Q = E(z)\sigma_{d\text{LT}}D/Q$$

Thus,

$$\text{SL}_{\text{annual}} = 1 - \frac{E(z)\sigma_{d\text{LT}}}{Q} \qquad (12\text{–}19)$$

EXAMPLE 12

Given a lead time service level of .90, $D = 1{,}000$, $Q = 250$, and $\sigma_{d\text{LT}} = 16$, determine (*a*) the annual service level, and (*b*) the amount of cycle safety stock that would provide an annual service level of .98. From Table 12.3, $E(z) = .048$ for a 90 percent lead time service level.

SOLUTION

a. Using Formula 12–19:

$\text{SL}_{\text{annual}} = 1 - .048(16)/250 = .997$

b. Using Formula 12–19 and an annual service level of .98, solve for $E(z)$:

$.98 = 1 - E(z)(16)/250$

Solving, $E(z) = .312$. From Table 12.3, with $E(z) = .312$, you can see that this value of $E(z)$ is a little more than the value of .307. So it appears that an acceptable value of z might be .19. The necessary safety stock to achieve the specified annual service level is equal to $z\sigma_{d\text{LT}}$. Hence, the safety stock is $.19(16) = 3.04$, or approximately 3 units.

Note that in the preceding example, a lead time service level of 90 percent provided an annual service level of 99.7 percent. Naturally, different values of D, Q, and $\sigma_{d\text{LT}}$ will tend to produce different results for a cycle service level of 90 percent. Nonetheless, the annual service level will usually be greater than the cycle service level. In addition, since the annual service level as defined relates to the percentage of units short per year, it makes sense to base cycle service levels on a specified annual service level. This means setting the annual level, using Formula 12–19 to solve for $E(z)$, and then using that value to obtain the service level for the order cycles.

Fill rate The percentage of demand filled by the stock on hand.

It should be mentioned that suppliers of service organizations are sometimes more concerned with the *fill rate* than with the number of units short. The **fill rate** is the percentage of demand filled by stock on hand.

HOW MUCH TO ORDER: FIXED-ORDER-INTERVAL MODEL

Fixed-order-interval (FOI) model Orders are placed at fixed time intervals.

The **fixed-order-interval (FOI) model** is used when orders must be placed at fixed time intervals (weekly, twice a month, etc.): The timing of orders is set. The question, then, at each order point, is how much to order. Fixed-interval ordering systems are widely used by retail businesses. If demand is variable, the order size will tend to vary from cycle to cycle. This is quite different from an EOQ/ROP approach in which the order size generally remains fixed from cycle to cycle, while the length of the cycle varies (shorter if demand is above average, and longer if demand is below average).

Reasons for Using the Fixed-Order-Interval Model

In some cases, a supplier's policy might encourage orders at fixed intervals. Even when that is not the case, grouping orders for items from the same supplier can produce savings in shipping costs. Furthermore, some situations do not readily lend themselves to continuous monitoring of inventory levels. Many retail operations (e.g., drugstores, small grocery stores) fall into this category. The alternative for them is to use fixed-interval ordering, which requires only periodic checks of inventory levels.

Determining the Amount to Order

If both the demand rate and lead time are constant, the fixed-interval model and the fixed-quantity model function identically. The differences in the two models become apparent only when examined under conditions of variability. Like the ROP model, the fixed-interval model can have variations in demand only, in lead time only, or in both demand and lead time. However, for the sake of simplicity and because it is perhaps the most frequently encountered situation, the discussion here will focus only on *variable demand* and *constant lead time.*

Figure 12.15 provides a comparison of the fixed-quantity and fixed-interval systems. In the fixed-quantity arrangement, orders are triggered by a *quantity* (ROP), while in the

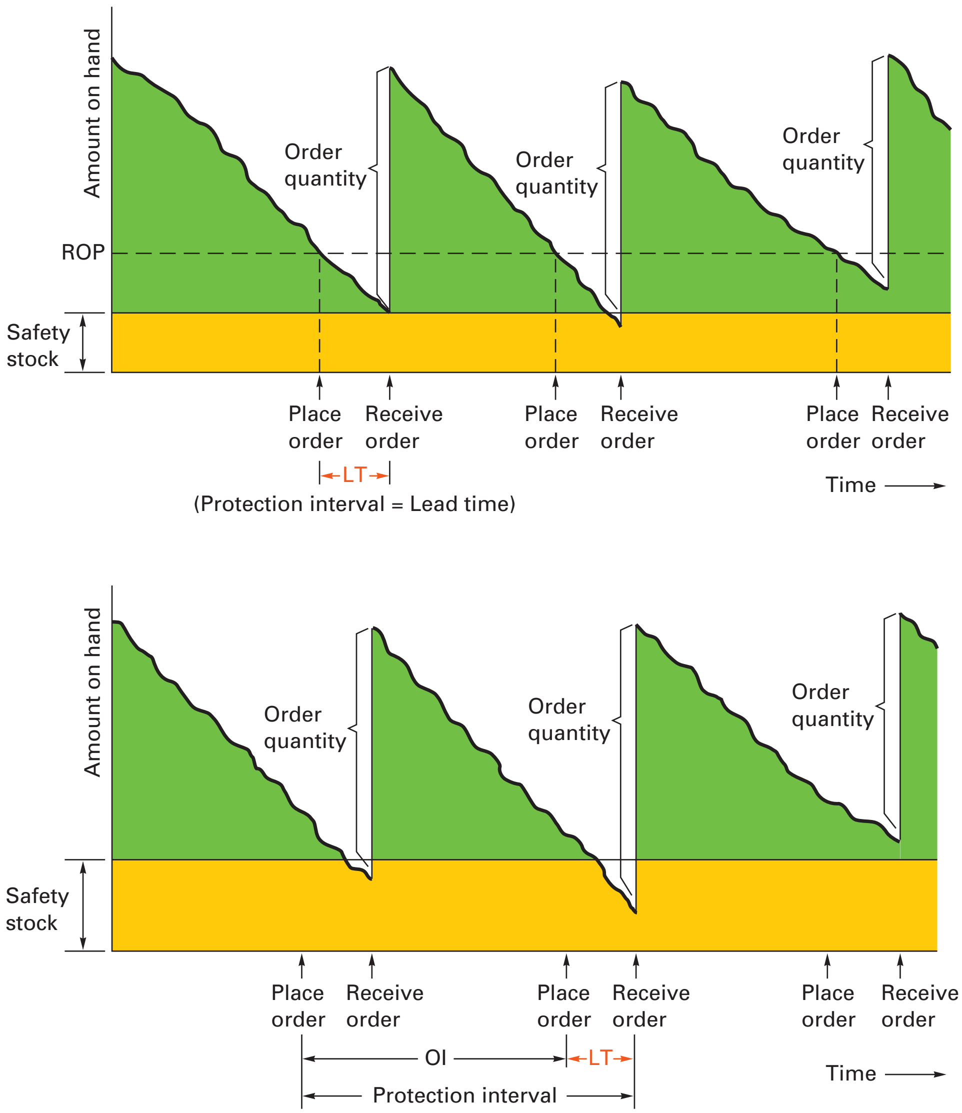


FIGURE 12.15
Comparison of fixed-quantity and fixed-interval ordering

A. Fixed quantity

B. Fixed interval

fixed-interval arrangement orders are triggered by a *time*. Therefore, the fixed-interval system must have stockout protection for lead time plus the next order cycle, but the fixed-quantity system needs protection only during lead time because additional orders can be placed at any time and will be received shortly (lead time) thereafter. Consequently, there is a greater need for safety stock in the fixed-interval model than in the fixed-quantity model. Note, for example, the large dip into safety stock during the second order cycle with the fixed-interval model.

Both models are sensitive to demand experience just prior to reordering, but in somewhat different ways. In the fixed-quantity model, a higher-than-normal demand causes a *shorter time* between orders, whereas in the fixed-interval model, the result is *a larger order size*. Another difference is that the fixed-quantity model requires close monitoring of inventory levels in order to know *when* the amount on hand has reached the reorder point. The fixed-interval model requires only a periodic review (i.e., physical count) of inventory levels just prior to placing an order to determine how much is needed.

Order size in the fixed-interval model is determined by the following computation:

$$\begin{aligned}\text{Amount to order} &= \text{Expected demand during protection interval} + \text{Safety stock} - \text{Amount on hand at reorder time} \\ &= \bar{d}(\text{OI}+\text{LT}) + z\sigma_d\sqrt{\text{OI}+\text{LT}} - A\end{aligned} \tag{12–20}$$

where

OI = Order interval (length of time between orders)

A = Amount on hand at reorder time

As in previous models, we assume that demand during the protection interval is normally distributed.

EXAMPLE 13

Given the following information, determine the amount to order.

$\bar{d}$ = 30 units per day	Desired service level = 99 percent
σ_d = 3 units per day	Amount on hand at reorder time = 71 units
LT = 2 days	OI = 7 days

SOLUTION

z = 2.33 for 99 percent service level

$$\begin{aligned}\text{Amount to order} &= \bar{d}(\text{OI}+\text{LT}) + z\sigma_d\sqrt{\text{OI}+\text{LT}} - A \\ &= 30(7+2) + 2.33(3)\sqrt{7+2} - 71 = 220 \text{ units}\end{aligned}$$

An issue related to fixed-interval ordering is the risk of a stockout. From the perspective (i.e., the point in time) of placing an order, there are two points in the order cycle at which a stockout could occur. One is shortly after the order is placed, while waiting to receive the current order (refer to Figure 12.15). The second point is near the end of the cycle, while waiting to receive the next order.

To find the initial risk of a stockout, use the ROP formula (12–13), setting ROP equal to the quantity on hand when the order is placed, and solve for z, then obtain the service level for that value of z from Appendix B, Table B and subtract it from 1.0000 to get the risk of a stockout.

To find the risk of a stockout at the end of the order cycle, use the fixed-interval formula (12–20) and solve for z. Then obtain the service level for that value of z from Appendix B, Table B and subtract it from 1.0000 to get the risk of a stockout.

Let's look at an example.

EXAMPLE 14

Given the following information:

$LT = 4$ days $\quad A = 43$ units

$OI = 12$ days $\quad Q = 171$ units

$\bar{d} = 10$ units/day

$\sigma_d = 2$ units/day

Determine the risk of a stockout at

a. The end of the initial lead time.

b. The end of the second lead time.

SOLUTION

a. For the risk of stockout for the first lead time, we use Formula 12–13. Substituting the given values, we get $43 = 10 \times 4 + z\,(2)(2)$. Solving, $z = +.75$. From Appendix B, Table B, the service level is .7734. The risk is $1 - .7734 = .2266$, which is fairly high.

b. For the risk of a stockout at the end of the second lead time, we use Formula 12–20. Substituting the given values we get $171 = 10 \times (4 + 12) + z\,(2)(4) - 43$. Solving, $z = +6.75$. This value is way out in the right tail of the normal distribution, making the service level virtually 100 percent, and, thus, the risk of a stockout at this point is essentially equal to zero.

Benefits and Disadvantages

The fixed-interval system results in tight control. In addition, when multiple items come from the same supplier, grouping orders can yield savings in ordering, packing, and shipping costs. Moreover, it may be the only practical approach if inventory withdrawals cannot be closely monitored.

On the negative side, the fixed-interval system necessitates a larger amount of safety stock for a given risk of stockout because of the need to protect against shortages during an entire order interval plus lead time (instead of lead time only), and this increases the carrying cost. Also, there are the costs of the periodic reviews.

THE SINGLE-PERIOD MODEL

The **single-period model** (sometimes referred to as the *newsboy problem*) is used to handle ordering of perishables (fresh fruits, vegetables, seafood, cut flowers) and items that have a limited useful life (newspapers, magazines, spare parts for specialized equipment). The *period* for spare parts is the life of the equipment, assuming that the parts cannot be used for other equipment. What sets unsold or unused goods apart is that they are not typically carried over from one period to the next, at least not without penalty. Day-old baked goods, for instance, are often sold at reduced prices; leftover seafood may be discarded; and out-of-date magazines may be offered to used book stores at bargain rates. There may even be some cost associated with disposal of leftover goods.

Single-period model Model for ordering of perishables and other items with limited useful lives.

Analysis of single-period situations generally focuses on two costs: shortage and excess. Shortage cost may include a charge for loss of customer goodwill as well as the opportunity cost of lost sales. Generally, **shortage cost** is simply unrealized profit per unit. That is,

Shortage cost Generally, the unrealized profit per unit.

$$C_{\text{shortage}} = C_s = \text{Revenue per unit} - \text{Cost per unit}$$

If a shortage or stockout relates to an item used in production or to a spare part for a machine, then shortage cost refers to the actual cost of lost production.

Excess cost pertains to items left over at the end of the period. In effect, excess cost is the difference between purchase cost and salvage value. That is,

Excess cost Difference between purchase cost and salvage value of items left over at the end of a period.

$$C_{\text{excess}} = C_e = \text{Original cost per unit} - \text{Salvage value per unit}$$

If there is cost associated with disposing of excess items, the salvage will be negative and will therefore *increase* the excess cost per unit.

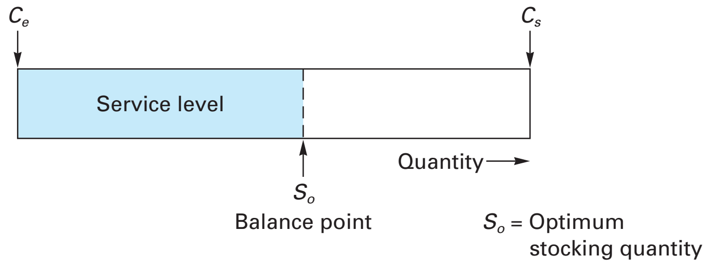


FIGURE 12.16
The optimal stocking level balances unit shortage and excess costs

The goal of the single-period model is to identify the order quantity, or stocking level, that will minimize the long-run excess and shortage costs.

There are two general categories of problems that we will consider: those for which demand can be approximated using a continuous distribution (perhaps a theoretical one such as a uniform or normal distribution) and those for which demand can be approximated using a discrete distribution (say, historical frequencies or a theoretical distribution such as the Poisson). The kind of inventory can indicate which type of model might be appropriate. For example, demand for petroleum, liquids, and gases tends to vary over some *continuous scale,* thus lending itself to description by a continuous distribution. Demand for tractors, cars, and computers is expressed in terms of the *number of units* demanded and lends itself to description by a discrete distribution.

Continuous Stocking Levels

The concept of identifying an optimal stocking level is perhaps easiest to visualize when demand is *uniform.* Choosing the stocking level is similar to balancing a seesaw, but instead of a person on each end of the seesaw, we have excess cost per unit (C_e) on one end of the distribution and shortage cost per unit (C_s) on the other. The optimal stocking level is analogous to the fulcrum of the seesaw; the stocking level equalizes the cost weights, as illustrated in Figure 12.16.

The *service level* is the *probability* that demand will not exceed the stocking level, and computation of the service level is the key to determining the optimal stocking level, S_o.

$$\text{Service level} = \frac{C_s}{C_s + C_e} \tag{12–21}$$

where

C_s = Shortage cost per unit
C_e = Excess cost per unit

If actual demand exceeds S_o, there is a shortage; hence, C_s is on the right end of the distribution. Similarly, if demand is less than S_o, there is an excess, so C_e is on the left end of the distribution. When $C_e = C_s$, the optimal stocking level is halfway between the endpoints of the distribution. If one cost is greater than the other, S_o will be closer to the larger cost.

EXAMPLE 15

Sweet cider is delivered weekly to Cindy's Cider Bar. Demand varies uniformly between 300 liters and 500 liters per week. Cindy pays 20 cents per liter for the cider and charges 80 cents per liter for it. Unsold cider has no salvage value and cannot be carried over into the next week due to spoilage. Find the optimal stocking level and its stockout risk for that quantity.

SOLUTION

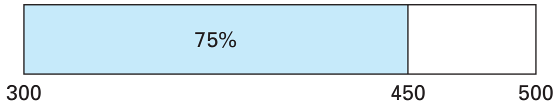
$$\begin{aligned} C_e &= \text{Cost per unit} - \text{Salvage value per unit} \\ &= \$0.20 \qquad\qquad - \$0 \\ &= \$0.20 \text{ per unit} \end{aligned}$$

$$C_s = \text{Revenue per unit} - \text{Cost per unit}$$
$$= \$0.80 - \$0.20$$
$$= \$0.60 \text{ per unit}$$

$$\text{SL} = \frac{C_s}{C_s + C_e} = \frac{\$0.60}{\$0.60 + \$0.20} = .75$$

Thus, the optimal stocking level must satisfy demand 75 percent of the time. For the uniform distribution, this will be at a point equal to the minimum demand plus 75 percent of the difference between maximum and minimum demands:

$$S_o = 300 + .75(500 - 300) = 450 \text{ liters}$$



The stockout risk is 1.00 − .75 = .25.

A similar approach applies when demand is normally distributed.

EXAMPLE 16

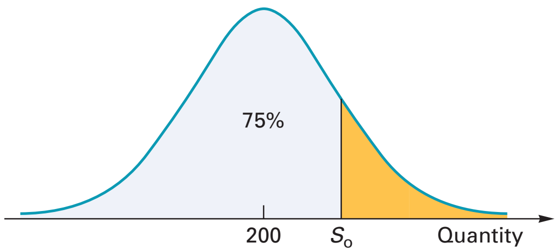
Cindy's Cider Bar also sells a blend of cherry juice and apple cider. Demand for the blend is approximately normal, with a mean of 200 liters per week and a standard deviation of 10 liters per week. C_s = 60 cents per liter, and C_e = 20 cents per liter. Find the optimal stocking level for the apple-cherry blend.

SOLUTION

$$\text{SL} = \frac{C_s}{C_s + C_e} = \frac{\$0.60}{\$0.60 + \$0.20} = .75$$

This indicates that 75 percent of the area under the normal curve must be to the left of the stocking level. Appendix B, Table B, shows that a value of z between +0.67 and +0.68, say, +0.675, will satisfy this. The optimal stocking level is S_o = mean + $z\sigma$. Thus,

$$S_o = 200 \text{ liters} + 0.675(10 \text{ liters}) = 206.75 \text{ liters}$$



Discrete Stocking Levels

When stocking levels are discrete rather than continuous, the service level computed using the ratio $C_s/(C_s + C_e)$ usually does not coincide with a feasible stocking level (e.g., the optimal amount may be *between* five and six units). The solution is to stock at the *next higher level* (e.g., six units). In other words, choose the stocking level so that the desired service level is equaled or *exceeded.* Figure 12.17 illustrates this concept.

The next example illustrates the use of an empirical distribution, followed by an example that illustrates the use of a Poisson distribution.

EXAMPLE 17

Historical records on the use of spare parts for several large hydraulic presses are to serve as an estimate of usage for spares of a newly installed press. Stockout costs involve downtime expenses and special ordering costs. These average \$4,200 per unit short. Spares cost \$800 each, and unused parts have zero salvage. Determine the optimal stocking level.

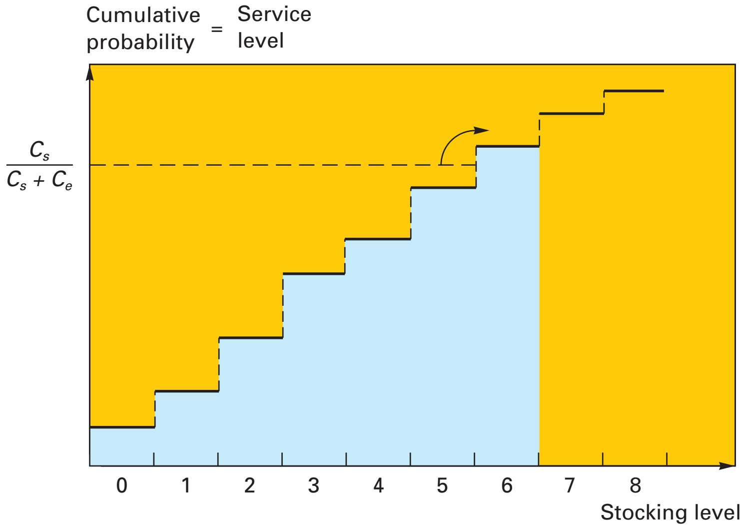


FIGURE 12.17
The service level achievement must equal or exceed the ratio $C_s/C_s + C_e$

Number of Spares Used	Relative Frequency	Cumulative Frequency
0	.20	.20
1	.40	.60
2	.30	.90
3	.10	1.00
4 or more	.00	
	1.00	

SOLUTION

$$C_s = \$4{,}200 \qquad C_e = \$800 \qquad \text{SL} = \frac{C_s}{C_s + C_e} = \frac{\$4{,}200}{\$4{,}200 + \$800} = .84$$

The cumulative-frequency column indicates the percentage of time that demand did not exceed (was equal to or less than) some amount. For example, demand does not exceed one spare 60 percent of the time, or two spares 90 percent of the time. Thus, in order to achieve a service level of *at least* 84 percent, it will be necessary to stock two spares (i.e., to go to the next higher stocking level).

EXAMPLE 18

Demand for long-stemmed red roses at a small flower shop can be approximated using a Poisson distribution that has a mean of four dozen per day. Profit on the roses is \$3 per dozen. Leftover flowers are marked down and sold the next day at a loss of \$2 per dozen. Assume that all marked-down flowers are sold. What is the optimal stocking level?

SOLUTION

$$C_s = \$3 \qquad C_e = \$2 \qquad \text{SL} = \frac{C_s}{C_s + C_e} = \frac{\$3}{\$3 + \$2} = .60$$

Obtain the cumulative frequencies from the Poisson table (Appendix B, Table C) for a mean of 4.0:

Demand (dozen per day)	Cumulative Frequency
0	.018
1	.092
2	.238
3	.433
4	.629
5	.785
⋮	⋮

Compare the service level to the cumulative frequencies. In order to attain a service level of at least .60, it is necessary to stock four dozen.

One final point about discrete stocking levels: If the computed service level is *exactly* equal to the cumulative probability associated with one of the stocking levels, there are *two* equivalent stocking levels in terms of minimizing long-run cost—the one with equal probability and the next higher one. In the preceding example, if the ratio had been equal to .629, we would be indifferent between stocking four dozen and stocking five dozen roses each day.

Operations Strategy

Inventories are a necessary part of doing business, but having too much inventory is not good. One reason is that inventories tend to hide problems; they make it easier to live with problems rather than eliminate them. Another reason is that inventories are costly to maintain. Consequently, a wise operations strategy is to work toward cutting back inventories by (1) reducing lot sizes and (2) reducing safety stocks.

One possibility for reducing the economic order quantity is to work to reduce the ordering cost, *S*. This might be accomplished by standardized procedures and perhaps by using *electronic data interchange* with suppliers. Another possibility is to examine holding cost, *H*. If this is understated, using a larger value will reduce the order quantity.

Japanese manufacturers use smaller lot sizes than their Western counterparts because they have a different perspective on inventory carrying costs. In addition to the usual components (e.g., storage, handling, obsolescence), the Japanese recognize the opportunity costs of disrupting the work flow, inability to place machines and workers closer together (which encourages cooperation, socialization, and communication), and hiding problems related to product quality and equipment breakdown. When these are factored in, carrying costs become higher—perhaps much higher—than before, and optimal lot sizes become smaller.

Companies may be able to achieve additional reductions in inventory by reducing the amount of safety stock carried. Important factors in safety stock are lead time and lead time variability, reductions of which will result in lower safety stocks. Firms can often realize these reductions by working with suppliers, choosing suppliers located close to the buyer, and shifting to smaller lot sizes.

The material presented in this chapter focuses primarily on management of *internal* inventories. However, successful inventory management also must include management of *external* inventories (i.e., inventory in the supply chain). Sharing demand data throughout the supply chain can alleviate the unnecessary buildup of safety stock in the supply chain that occurs when information isn't shared. Manufacturers and suppliers can judge the timing of orders from customers, and customers can use information about suppliers' inventories to set reasonable lead times.

Last, it is important to make sure that inventory records be kept *accurate* and *up-to-date*. Estimates of holding costs, ordering costs, setup costs, and lead times should be reviewed periodically and updated as necessary.

SUMMARY

Inventory management is a core operations management activity. Good inventory management is often the mark of a well-run organization. Inventory levels must be planned carefully in order to balance the cost of holding inventory and the cost of providing reasonable levels of customer service. Successful inventory management requires a system to keep track of inventory transactions, accurate information about demand and lead times, realistic estimates of certain inventory-related costs, and a priority system for classifying the items in inventory and allocating control efforts.

Four classes of models are described: EOQ, ROP, fixed-order-interval, and single-period models. The first three are appropriate if unused items can be carried over into subsequent periods. The single-period model is appropriate when items cannot be carried over. EOQ models address the question of how much to order. The ROP models address the question of when to order and are particularly helpful in dealing with situations that include variations in either demand rate or lead time. ROP models involve service level and safety stock considerations. When the time between orders is fixed, the FOI model is useful. The single-period model is used for items that have a "shelf life" of one period. The models presented in this chapter are summarized in Table 12.4.

TABLE 12.4 Summary of inventory formulas

Model	Formula	Symbols
1. Basic EOQ	$Q_0 = \sqrt{\frac{2DS}{H}}$ (12–2) $TC = \frac{Q}{2}H + \frac{D}{Q}S$ (12–1) Length of order cycle $= \frac{Q}{D}$ (12–3)	Q_o = Economic order quantity D = Annual demand S = Order cost H = Annual carrying cost per unit Q = Order quantity
2. Economic production quantity	$Q_o = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}}$ (12–5) $TC = \frac{I_{max}}{2}H + \frac{D}{Q}S$ (12–4) Cycle time $= \frac{Q}{u}$ (12–6) Run time $= \frac{Q}{p}$ (12–7) $I_{max} = \frac{Q_0}{p}(p-u)$ (12–8)	Q_o = Optimal run or order size p = Production or delivery rate u = Usage rate I_{MAX} = Maximum inventory level
3. Quantity discounts	$TC = \frac{Q}{2}H + \frac{D}{Q}S + PD$ (12–9)	P = Unit price
4. Reorder point under: a. Constant demand and lead time b. Variable demand rate c. Variable lead time d. Variable lead time and demand	 $ROP = d(LT)$ (12–10) $ROP = \bar{d}LT + z(\sigma_d)\sqrt{LT}$ (12–13) $ROP = d\overline{LT} + z(\sigma_{LT})d$ (12–14) $ROP = \bar{d}\overline{LT} + z\sqrt{\overline{LT}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$ (12–15)	ROP = Quantity on hand at reorder point d = Demand rate LT = Lead time $\bar{d}$ = Average demand rate σ_d = Standard deviation of demand rate z = Standard normal deviation $\overline{LT}$ = Average lead time σ_{LT} = Standard deviation of lead time
5. ROP shortages a. Units short per cycle b. Units short per year c. Annual service level	$E(n) = E(z)\sigma_{dLT}$ (12–16) $E(N) = E(n)\frac{D}{Q}$ (12–17) $SL_{annual} = 1 - \frac{E(z)\sigma_{dLT}}{Q}$ (12–19)	$E(n)$ = Expected number short per cycle $E(z)$ = Standardized number short σ_{dLT} = Standard deviation of lead time demand $E(N)$ = Expected number short per year SL_{annual} = Annual service level
6. Fixed interval	$Q = \bar{d}(OI + LT) + z\sigma_d\sqrt{OI + LT} - A$ (12–20)	OI = Time between orders A = Amount on hand at order time
7. Single period	$SL = \frac{C_s}{C_s + C_e}$ (12–21)	SL = Service level C_s = Shortage cost per unit C_e = Excess cost per unit

KEY TERMS

A-B-C approach, 548
cycle counting, 549
economic order quantity (EOQ), 550
excess cost, 573
fill rate, 570
fixed-order-interval (FOI) model, 570
holding (carrying) cost, 547
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lead time, 547
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ordering costs, 548
periodic system, 545
perpetual inventory system, 545
point-of-sale (POS) system, 547
quantity discounts, 558
reorder point (ROP), 563

SOLVED PROBLEMS

Problem 1

Basic EOQ. A toy manufacturer uses approximately 32,000 silicon chips annually. The chips are used at a steady rate during the 240 days a year that the plant operates. Annual holding cost is \$3 per chip, and ordering cost is \$120. Determine

a. The optimal order quantity.

b. The number of workdays in an order cycle.

Solution

D = 32,000 chips per year S = \$120

H = \$3 per unit per year

a. $Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(32{,}000)120}{3}} = 1{,}600$ chips.

b. $\frac{Q}{D} = \frac{1{,}600 \text{ chips}}{32{,}000 \text{ chips/yr.}} = \frac{1}{20}$ year (i.e., $\frac{1}{20} \times 240$ days), or 12 days.

Problem 2

Economic production quantity. The Dine Corporation is both a producer and a user of brass couplings. The firm operates 220 days a year and uses the couplings at a steady rate of 50 per day. Couplings can be produced at a rate of 200 per day. Annual storage cost is \$2 per coupling, and machine setup cost is \$70 per run.

a. Determine the economic run quantity.

b. Approximately how many runs per year will there be?

c. Compute the maximum inventory level.

d. Determine the length of the *pure consumption* portion of the cycle.

Solution

D = 50 units per day × 220 days per year = 11,000 units per year

S = \$70

H = \$2 per unit per year

p = 200 units per day

u = 50 units per day

a. $Q_0 = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(11{,}000)70}{2}}\sqrt{\frac{200}{200-50}} \approx 1{,}013$ units.

b. Number of runs per year: $D/Q_0 = 11{,}000/1{,}013 = 10.86$, or approximately 11.

c. $I_{max} = \frac{Q_0}{p}(p-u) = \frac{1{,}013}{200}(200-50) = 759.75$ or 760 units.

d. $\text{Length of cycle} = \frac{Q}{u} = \frac{1{,}013 \text{ units}}{50 \text{ units per day}} = 20.26$ days

$\text{Length of run} = \frac{Q}{p} = \frac{1{,}013 \text{ units}}{200 \text{ units per day}} = 5.065$ days

Length of pure consumption portion = Length of cycle − Length of run

= 20.26 − 5.065 = 15.20 days.

Problem 3

Quantity discounts. A small manufacturing firm uses roughly 3,400 pounds of chemical dye a year. Currently the firm purchases 300 pounds per order and pays \$3 per pound. The supplier has just announced that orders of 1,000 pounds or more will be filled at a price of \$2 per pound. The manufacturing firm incurs a cost of \$100 each time it submits an order and assigns an annual holding cost of 17 percent of the purchase price per pound.

a. Determine the order size that will minimize the total cost.

b. If the supplier offered the discount at 1,500 pounds instead of 1,000 pounds, what order size would minimize total cost?

Solution

D = 3,400 pounds per year $\quad S = \$100 \quad H = 0.17P$

a. Compute the EOQ for \$2 per pound:

The quantity ranges are

Range	Unit Price
1 to 999	\$3
1,000+	\$2

$$Q_{\$2/\text{pound}} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(3{,}400)100}{0.17(2)}} = 1{,}414 \text{ pounds}$$

Because this quantity is feasible at \$2 per pound, it is the optimum.

b. When the discount is offered at 1,500 pounds, the EOQ for the \$2 per pound range is no longer feasible. Consequently, it becomes necessary to compute the EOQ for \$3 per pound and compare the total cost for that order size with the total cost using the price break quantity (i.e., 1,500).

$$Q_{\$3/\text{pound}} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(3{,}400)100}{0.17(3)}} \approx 1{,}155 \text{ pounds}$$

$$\text{TC} = \left(\frac{Q}{2}\right)H + \left(\frac{D}{Q}\right)S + PD$$

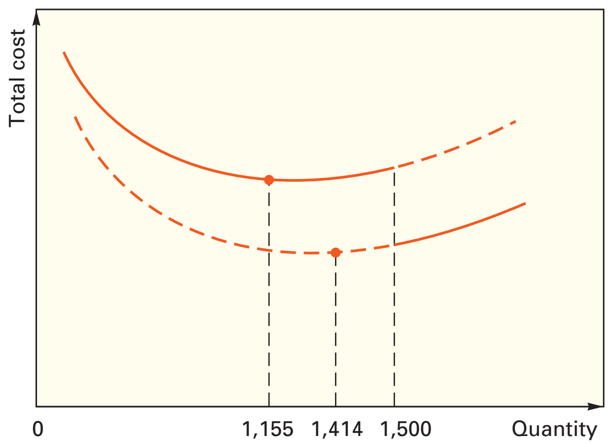
$$\text{TC}_{1,155} = \left(\frac{1{,}155}{2}\right)0.17(3) + \left(\frac{3{,}400}{1{,}155}\right)100 + 3(3{,}400)$$

$$= \$294.53 + \$294.37 + \$10{,}200 = \$10{,}789$$

$$\text{TC}_{1,500} = \left(\frac{1{,}500}{2}\right)0.17(2) + \left(\frac{3{,}400}{1{,}500}\right)100 + 2(3{,}400)$$

$$= \$255 + \$226.67 + \$6{,}800 = \$7{,}282$$

Hence, because it would result in a lower total cost, 1,500 is the optimal order size.



Problem 4

ROP for variable demand and constant lead time. The housekeeping department of a motel uses approximately 400 washcloths per day. The actual number tends to vary with the number of guests on any given night. Usage can be approximated by a normal distribution that has a mean of 400 and a standard deviation of 9 washcloths per day. A linen supply company delivers towels and washcloths with a lead time of three days. If the motel policy is to maintain a stockout risk of 2 percent, what is the minimum number of washcloths that must be on hand at reorder time, and how much of that amount can be considered safety stock?

Solution

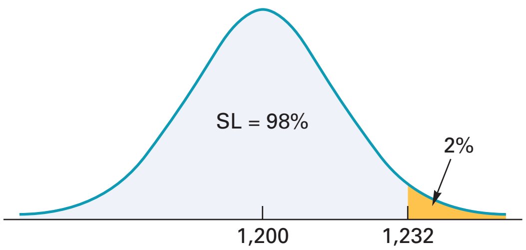
$\bar{d}$ = 400 washcloths per day LT = 3 days

σ_d = 9 washcloths per day Risk = 2 percent, so service level = 98 percent

From Appendix B, Table B, the z value that corresponds to an area under the normal curve to the left of z for 98 percent is about +2.055.

$$\text{ROP} = \bar{d}\text{LT} + z\sigma_d\sqrt{\text{LT}} = 400(3) + 2.055(9)\sqrt{3}$$
$$= 1{,}200 + 32.03, \text{ or approximately } 1{,}232 \text{ washcloths}$$

Safety stock is approximately 32 washcloths.



Problem 5

ROP for constant demand and variable lead time. The motel in the preceding example uses approximately 600 bars of soap each day, and this tends to be fairly constant. Lead time for soap delivery is normally distributed with a mean of six days and a standard deviation of two days. A service level of 90 percent is desired.

a. Find the ROP.

b. How many days of supply are on hand at the ROP?

Solution

d = 600 bars per day

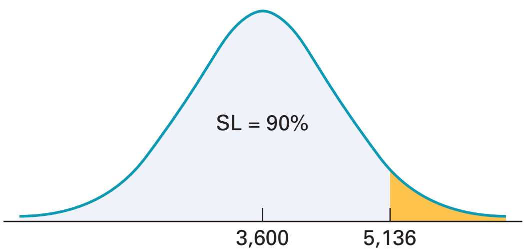
SL = 90 percent, so z = +1.28 (from Appendix B, Table B)

$\overline{\text{LT}}$ = 6 days

σ_{LT} = 2 days

a. $\text{ROP} = d\,\overline{\text{LT}} + z(\sigma_{\text{LT}})d = 600(6) + 1.28(2)(600)$
$= 5{,}136$ bars of soap

b. $\dfrac{\text{ROP}}{d} = \dfrac{5{,}136}{600} = 8.56$ days



Problem 6

ROP for variable demand rate and variable lead time. The motel replaces broken glasses at a rate of 25 per day. In the past, this quantity has tended to vary normally and have a standard deviation of 3 glasses per day. Glasses are ordered from a Cleveland supplier. Lead time is normally distributed with an average of 10 days and a standard deviation of 2 days. What ROP should be used to achieve a service level of 95 percent?

Solution

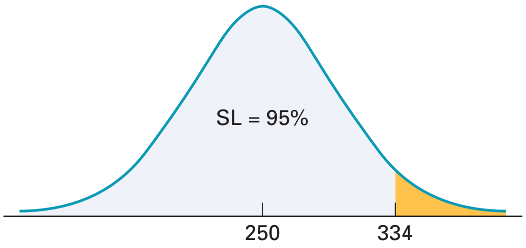
$\bar{d}$ = 25 glasses per day $\overline{\text{LT}}$ = 10 days

σ_d = 3 glasses per day σ_{LT} = 2 days

SL = 95 percent, so $z = +1.65$ (Appendix B, Table B)

$$\text{ROP} = \bar{d}\,\overline{\text{LT}} + z\sqrt{\overline{\text{LT}}\sigma_d^2 + \bar{d}\sigma_{\text{LT}}^2}$$

$$= 25(10) + 1.65\sqrt{10(3)^2 + (25)^2(2)^2} = 334 \text{ glasses}$$



Problem 7

Shortages and service levels. The manager of a store that sells office supplies has decided to set an annual service level of 96 percent for a certain model of telephone answering equipment. The store sells approximately 300 of this model a year. Holding cost is $5 per unit annually, ordering cost is $25, and $\sigma_{d\text{LT}} = 7$.

a. What average number of units short per year will be consistent with the specified annual service level?

b. What average number of units short per cycle will provide the desired annual service level?

c. What lead time service level is necessary for the 96 percent annual service level?

Solution

$\text{SL}_{\text{annual}} = 96$ percent $D = 300$ units $H = \$5$ $S = \$25$ $\sigma_{d\text{LT}} = 7$

a. $E(N) = (1 - \text{SL}_{\text{annual}})\,D = (1 - .96)(300) = 12$ units

b. $E(N) = E(n)\dfrac{D}{Q}$. Solving for $E(n)$, you have

$$E(n) = E(N) \div \left(\frac{D}{Q}\right) = 12 \div \left(\frac{300}{Q}\right)$$

$$Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(300)(25)}{5}} = 54.77 \text{ (round to 55)}$$

Then $E(n) = 12 \div \left(\dfrac{300}{55}\right) = 2.2.$

c. In order to find the lead time service level, you need the value of $E(z)$. Because the value of $E(n)$ is 2.2 and $E(n) = E(z)\sigma_{d\text{LT}}$, you have $2.2 = E(z)(7)$. Solving gives $E(z) = 2.2 \div 7 = 0.314$. Interpolating in Table 12.3 gives the approximate lead time service level. Thus,

$$\frac{.307 - .314}{.307 - .324} = \frac{.5793 - x}{.5793 - .5636}$$

Solving,

$x = .5728$

[To interpolate, find the two values between which the computed number falls in the $E(z)$ column. Then find the difference between the computed value and one end of the range, and divide by the difference between the two ends of the range. Perform the corresponding calculation on the two service levels using x for the unknown value, and solve for x. Often, simply "eyeballing" the unknown value will suffice.]

Problem 8

Fixed-order-interval model. A lab orders a number of chemicals from the same supplier every 30 days. Lead time is five days. The assistant manager of the lab must determine how much of one of these chemicals to order. A check of stock revealed that eleven 25-milliliter (ml) jars are on hand. Daily usage of the chemical is approximately normal with a mean of 15.2 ml per day and a standard deviation of 1.6 ml per day. The desired service level for this chemical is 95 percent.

a. How many jars of the chemical should be ordered?
b. What is the average amount of safety stock of the chemical?

Solution

$\bar{d}$ = 15.2 ml per day OI = 30 days SL = 95% requires $z = 1.65$

σ_d = 1.6 ml per day LT = 5 days A = 11 jars × 25 ml per jar = 275 ml

a. Amount to order $= \bar{d}(\text{OI} + \text{LT}) + z\sigma_d\sqrt{\text{OI} + \text{LT}} - A$

$$= 15.2(30 + 5) + 1.65(1.6)\sqrt{30 + 5} - 275 = 272.62 \text{ ml}$$

Convert this to number of jars:

$$\frac{272.62 \text{ ml}}{25 \text{ ml per jar}} = 10.90 \text{ or } 11 \text{ jars}$$

b. Safety stock $= z\sigma_d\sqrt{\text{OI} + \text{LT}} = 1.65(1.6)\sqrt{30 + 5} = 15.62$ ml.

Problem 9

Single-period model. A firm that installs cable TV systems uses a certain piece of equipment for which it carries two spare parts. The parts cost $500 each and have no salvage value. Part failures can be modeled by a Poisson distribution with a mean of two failures during the useful life of the equipment. Holding and disposal costs are negligible. Estimate the apparent range of shortage cost.

Solution

C_s is unknown $C_e = \$500$

The Poisson table (Appendix B, Table C) provides these values for a mean of 2.0:

Number of Failures	Cumulative Probability
0	.135
1	.406
2	.677
3	.857
4	.947
5	.983
⋮	⋮

For the optimal stocking level, the service level must usually be rounded up to a feasible stocking level. Hence, you know that the service level must have been between .406 and .677 in order to make two units the optimal level. By setting the service level equal first to .406 and then to .677, you can establish bounds on the possible range of shortage costs.

$$\frac{C_s}{C_s + \$500} = .406, \text{ so } C_s = .406(\$500 + C_s)$$

Solving, you find $C_s = \$341.75$.

Similarly,

$$\frac{C_s}{C_s + \$500} = .677, \text{ so } C_s = .677(\$500 + C_s)$$

Solving, you find $C_s = \$1{,}047.99$. Hence, the apparent range of shortage cost is $341.75 to $1,047.99.

DISCUSSION AND REVIEW QUESTIONS

1. What are the primary reasons for holding inventory?
2. What are the requirements for effective inventory management?
3. Briefly describe each of the costs associated with inventory.
4. What potential benefits and risks do RFID tags have for inventory management?
5. Why might it be inappropriate to use inventory turnover ratios to compare inventory performance of companies that are in different industries?
6. List the major assumptions of the EOQ model.
7. How would you respond to the criticism that EOQ models tend to provide misleading results because values of *D, S,* and *H* are, at best, educated guesses?
8. Explain briefly how a higher carrying cost can result in a decrease in inventory.
9. What is safety stock, and what is its purpose?
10. Under what circumstances would the amount of safety stock held be
    a. Large? b. Small? c. Zero?
11. What is meant by the term *service level?* Generally speaking, how is service level related to the amount of safety stock held?
12. Describe briefly the A-B-C approach to inventory control.
13. The purchasing agent for a company that assembles and sells air-conditioning equipment in a Latin American country has noted that the cost of compressors has increased significantly each time they have reordered. The company uses an EOQ model to determine order size. What are the implications of this price escalation with respect to order size? What factors other than price must be taken into consideration?
14. Explain how a decrease in setup time can lead to a decrease in the average amount of inventory a firm holds, and why that would be beneficial.
15. What is the single-period model, and under what circumstances is it appropriate?
16. Can the optimal stocking level in the single-period model ever be less than expected demand? Explain briefly.
17. What are some ways in which a company can reduce the need for inventories?

TAKING STOCK

1. What trade-offs are involved in each of these aspects of inventory management?
   a. Buying additional amounts to take advantage of quantity discounts.
   b. Treating holding cost as a percentage of unit price instead of as a constant amount.
   c. Conducting cycle counts once a quarter instead of once a year.
2. Who needs to be involved in inventory decisions involving holding costs? Setting inventory levels? Quantity discount purchases?
3. How has technology aided inventory management? How have technological improvements in products such as automobiles and computers impacted inventory decisions?

CRITICAL THINKING EXERCISES

1. To be competitive, many fast-food chains began to expand their menus to include a wider range of foods. Although contributing to competitiveness, this has added to the complexity of operations, including inventory management. Specifically, in what ways does the expansion of menu offerings create problems for inventory management?
2. As a supermarket manager, how would you go about evaluating the criticalness of an inventory shortage?

EXPERIENTIAL LEARNING EXERCISES

1. Describe two incidents in your personal life that were affected by inventory and the way you were affected. For instance, when you tried to purchase a food item (perhaps an ingredient in a recipe) and discovered the store was out of that item, or an article of clothing you wanted but the store didn't have your size, or items that you discarded (food that was no longer fresh), an article of clothing you "had" to have, but then rarely if ever wore.
2. a. For each of the following products that you purchase or use, identify the ordering system(s) that you employ and state what factor or factors guide your quantity decision.

      Magazines, fuel for your vehicle, printer ink, shampoo, candy or other snacks, fresh fruit.

   b. For each of the following services you use, identify the ordering system(s) you employ (i.e., how do you decide on the timing?).

      Haircut, car wash, a dry cleaner or laundromat, food delivery, video rental.

PROBLEMS

1. The manager of an automobile repair shop hopes to achieve a better allocation of inventory control efforts by adopting an A-B-C approach to inventory control. Given the monthly usages in the following table, classify the items in A, B, and C categories according to dollar usage:

Item	Usage	Unit Cost
4021 . . .	50	$1,400
9402 . . .	300	12
4066 . . .	40	700
6500 . . .	150	20
9280 . . .	10	1,020
4050 . . .	80	140
6850 . . .	2,000	15
3010 . . .	400	20
4400 . . .	7,000	5

2. The following table contains figures on the monthly volume and unit costs for a random sample of 16 items from a list of 2,000 inventory items at a health care facility:

Item	Unit Cost	Usage
K34	$10	200
K35	25	600
K36	36	150
M10	16	25
M20	20	80
Z45	80	200
F14	20	300
F95	30	800
F99	20	60
D45	10	550
D48	12	90
D52	15	110
D57	40	120
N08	30	40
P05	16	500
P09	10	30

a. Develop an A-B-C classification for these items.
b. How could the manager use this information?
c. After reviewing your classification scheme, suppose that the manager decides to place item P05 into the A category. What are some possible explanations for this decision?

3. A large bakery buys flour in 25-pound bags. The bakery uses an average of 4,860 bags a year. Preparing an order and receiving a shipment of flour involves a cost of $10 per order. Annual carrying costs are $75 per bag.
a. Determine the economic order quantity.
b. What is the average number of bags on hand?
c. How many orders per year will there be?
d. Compute the total cost of ordering and carrying flour.
e. If ordering costs were to increase by $1 per order, how much would that affect the minimum total annual cost?

4. A large law firm uses an average of 40 boxes of copier paper a day. The firm operates 260 days a year. Storage and handling costs for the paper are $30 a year per box, and it costs approximately $60 to order and receive a shipment of paper.
a. What order size would minimize the sum of annual ordering and carrying costs?
b. Compute the total annual cost using your order size from part *a*.
c. Except for rounding, are annual ordering and carrying costs always equal at the EOQ?
d. The office manager is currently using an order size of 200 boxes. The partners of the firm expect the office to be managed "in a cost-efficient manner." Would you recommend that the office manager use the optimal order size instead of 200 boxes? Justify your answer.

5. Garden Variety Flower Shop uses 750 clay pots a month. The pots are purchased at $2 each. Annual carrying costs per pot are estimated to be 30 percent of cost, and ordering costs are $20 per order. The manager has been using an order size of 1,500 flower pots.
   a. What additional annual cost is the shop incurring by staying with this order size?
   b. Other than cost savings, what benefit would using the optimal order quantity yield?

6. A produce distributor uses 800 packing crates a month, which it purchases at a cost of $10 each. The manager has assigned an annual carrying cost of 35 percent of the purchase price per crate. Ordering costs are $28. Currently the manager orders once a month. How much could the firm save annually in ordering and carrying costs by using the EOQ?

7. A manager receives a forecast for next year. Demand is projected to be 600 units for the first half of the year and 900 units for the second half. The monthly holding cost is $2 per unit, and it costs an estimated $55 to process an order.
   a. Assuming that monthly demand will be level during each of the six-month periods covered by the forecast (e.g., 100 per month for each of the first six months), determine an order size that will minimize the sum of ordering and carrying costs for each of the six-month periods.
   b. Why is it important to be able to assume that demand will be level during each six-month period?
   c. If the vendor is willing to offer a discount of $10 *per order* for ordering in multiples of 50 units (e.g., 50, 100, 150), would you advise the manager to take advantage of the offer in either period? If so, what order size would you recommend?

8. A food processor uses approximately 27,000 glass jars a month for its fruit juice product. Because of storage limitations, a lot size of 4,000 jars has been used. Monthly holding cost is 18 cents per jar, and reordering cost is $60 per order. The company operates an average of 20 days a month.
   a. What penalty is the company incurring by its present order size?
   b. The manager would prefer ordering 10 times each month but would have to justify any change in order size. One possibility is to simplify order processing to reduce the ordering cost. What ordering cost would enable the manager to justify ordering every other day?
   c. Suppose that after investigating ordering cost, the manager is able to reduce it to $50. How else could the manager justify using an order size that would be consistent with ordering every other day?

9. The Friendly Sausage Factory (FSF) can produce hot dogs at a rate of 5,000 per day. FSF supplies hot dogs to local restaurants at a steady rate of 250 per day. The cost to prepare the equipment for producing hot dogs is $66. Annual holding costs are 45 cents per hot dog. The factory operates 300 days a year. Find
   a. The optimal run size.
   b. The number of runs per year.
   c. The length (in days) of a run.

10. A chemical firm produces sodium bisulfate in 100-pound bags. Demand for this product is 20 tons per day. The capacity for producing the product is 50 tons per day. Setup costs $100, and storage and handling costs are $5 per ton a year. The firm operates 200 days a year. (Note: 1 ton = 2,000 pounds.)
   a. How many bags per run are optimal?
   b. What would the average inventory be for this lot size?
   c. Determine the approximate length of a production run, in days.
   d. About how many runs per year would there be?
   e. How much could the company save annually if the setup cost could be reduced to $25 per run?

11. A company is about to begin production of a new product. The manager of the department that will produce one of the components for the product wants to know how often the machine used to produce the item will be available for other work. The machine will produce the item at a rate of 200 units a day. Eighty units will be used daily in assembling the final product. Assembly will take place five days a week, 50 weeks a year. The manager estimates that it will take almost a full day to get the machine ready for a production run, at a cost of $300. Inventory holding costs will be $10 a year.
   a. What run quantity should be used to minimize total annual costs?
   b. What is the length of a production run in days?
   c. During production, at what rate will inventory build up?

d. If the manager wants to run another job between runs of this item, and needs a minimum of 10 days per cycle for the other work, will there be enough time?
e. Given your answer to part *d*, the manager wants to explore options that will allow this other job to be performed using this equipment. Name three options the manager can consider.
f. Suppose the manager decides to increase the run size of the new product. How many additional units would be needed to just accommodate the other job? How much will that increase the total annual cost?

12. A company manufactures hair dryers. It buys some of the components, but it makes the heating element, which it can produce at the rate of 800 per day. Hair dryers are assembled daily, 250 days a year, at a rate of 300 per day. Because of the disparity between the production and usage rates, the heating elements are periodically produced in batches of 2,000 units.
a. Approximately how many *batches* of heating elements are produced annually?
b. If production on a batch begins when there is no inventory of heating elements on hand, how much inventory will be on hand *two days later?*
c. What is the average inventory of elements, assuming each production cycle begins when there are none on hand?
d. The same equipment that is used to make the heating elements could also be used to make a component for another of the firm's products. That job would require four days, including setup. Setup time for making a batch of the heating elements is a half day. Is there enough time to do this job between production of batches of heating elements? Explain.

13. A mail-order house uses 18,000 boxes a year. Carrying costs are 60 cents per box a year, and ordering costs are $96. The following price schedule applies. Determine
a. The optimal order quantity.
b. The number of orders per year.

Number of Boxes	Price per Box
1,000 to 1,999	$1.25
2,000 to 4,999	1.20
5,000 to 9,999	1.15
10,000 or more	1.10

14. A jewelry firm buys semiprecious stones to make bracelets and rings. The supplier quotes a price of $8 per stone for quantities of 600 stones or more, $9 per stone for orders of 400 to 599 stones, and $10 per stone for lesser quantities. The jewelry firm operates 200 days per year. Usage rate is 25 stones per day, and ordering costs are $48.
a. If carrying costs are $2 per year for each stone, find the order quantity that will minimize total annual cost.
b. If annual carrying costs are 30 percent of unit cost, what is the optimal order size?
c. If lead time is six working days, at what point should the company reorder?

15. A manufacturer of exercise equipment purchases the pulley section of the equipment from a supplier who lists these prices: less than 1,000, $5 each; 1,000 to 3,999, $4.95 each; 4,000 to 5,999, $4.90 each; and 6,000 or more, $4.85 each. Ordering costs are $50, annual carrying costs per unit are 40 percent of purchase cost, and annual usage is 4,900 pulleys. Determine an order quantity that will minimize total cost.

16. A company will begin stocking remote control devices. Expected monthly demand is 800 units. The controllers can be purchased from either supplier A or supplier B. Their price lists are as follows:

SUPPLIER A		SUPPLIER B	
Quantity	**Unit Price**	**Quantity**	**Unit Price**
1–199	$14.00	1–149	$14.10
200–499	13.80	150–349	13.90
500+	13.60	350+	13.70

Ordering cost is $40 and annual holding cost is 25 percent of unit price per unit. Which supplier should be used and what order quantity is optimal if the intent is to minimize total annual costs?

17. A manager just received a new price list from a supplier. It will now cost $1.00 a box for order quantities of 801 or more boxes, $1.10 a box for 200 to 800 boxes, and $1.20 a box for smaller quantities. Ordering cost is $80 per order and carrying costs are $10 per box a year. The firm uses 3,600 boxes a year. The manager has suggested a "round number" order size of 800 boxes. The manager's rationale is that with a U-shaped cost curve that is fairly flat at its minimum, the difference in total annual cost between 800 and 801 units would be small anyway. How would you reply to the manager's suggestion? What order size would you recommend?

18. A newspaper publisher uses roughly 800 feet of baling wire each day to secure bundles of newspapers while they are being distributed to carriers. The paper is published Monday through Saturday. Lead time is six workdays. What is the appropriate reorder point quantity, given that the company desires a service level of 95 percent, if that stockout risk for various levels of safety stock are as follows: 1,500 feet, 0.10; 1,800 feet, 0.05; 2,100 feet, 0.02; and 2,400 feet, 0.01?

19. Given this information:

Expected demand during lead time = 300 units

Standard deviation of lead time demand = 30 units

Determine each of the following, assuming that lead time demand is distributed normally:

a. The ROP that will provide a risk of stockout of 1 percent during lead time.
b. The safety stock needed to attain a 1 percent risk of stockout during lead time.
c. Would a stockout risk of 2 percent require more or less safety stock than a 1 percent risk? Explain. Would the ROP be larger, smaller, or unaffected if the acceptable risk was 2 percent instead of 1 percent? Explain.

20. Given this information:

Lead-time demand = 600 pounds

Standard deviation of lead-time demand = 52 pounds (Assume normality.)

Acceptable stockout risk during lead time = 4 percent

a. What amount of safety stock is appropriate?
b. When should this item be reordered?
c. What risk of stockout would result from a decision not to have any safety stock?

21. Demand for walnut fudge ice cream at the Sweet Cream Dairy can be approximated by a normal distribution with a mean of 21 gallons per week and a standard deviation of 3.5 gallons per week. The new manager desires a service level of 90 percent. Lead time is two days, and the dairy is open seven days a week. (Hint: Work in terms of weeks.)

a. If an ROP model is used, what ROP would be consistent with the desired service level? How many days of supply are on hand at the ROP, assuming average demand?
b. If a fixed-interval model is used instead of an ROP model, what order size would be needed for the 90 percent service level with an order interval of 10 days and a supply of 8 gallons on hand at the order time? What is the probability of experiencing a stockout before this order arrives?
c. Suppose the manager is using the ROP model described in part *a*. One day after placing an order with the supplier, the manager receives a call from the supplier that the order will be delayed because of problems at the supplier's plant. The supplier promises to have the order there in two days. After hanging up, the manager checks the supply of walnut fudge ice cream and finds that 2 gallons have been sold since the order was placed. Assuming the supplier's promise is valid, what is the probability that the dairy will run out of this flavor before the shipment arrives?

22. The injection molding department of a company uses an average of 30 gallons of special lubricant a day. The supply of the lubricant is replenished when the amount on hand is 170 gallons. It takes four days for an order to be delivered. Safety stock is 50 gallons, which provides a stockout risk of 9 percent. What amount of safety stock would provide a stockout risk of 3 percent? Assume normality.

23. A company uses 85 circuit boards a day in a manufacturing process. The person who orders the boards follows this rule: Order when the amount on hand drops to 625 boards. Orders are delivered approximately six days after being placed. The delivery time is normal with a mean of six days and a standard deviation of 1.10 days. What is the probability that the supply of circuit boards will be exhausted before the order is received if boards are reordered when the amount on hand drops to 625 boards?

24. One item a computer store sells is supplied by a vendor who handles only that item. Demand for that item recently changed, and the store manager must determine when to replenish it. The manager wants a probability of at least 96 percent of not having a stockout during lead time. The manager expects demand to average a dozen units a day and have a standard deviation of 2 units a day. Lead time is variable, averaging four days with a standard deviation of one day. Assume normality and that seasonality is not a factor.

a. When should the manager reorder to achieve the desired probability?
b. Why might the model not be appropriate if seasonality was present?

25. The manager of a car wash received a revised price list from the vendor who supplies soap, and a promise of a shorter lead time for deliveries. Formerly the lead time was four days, but now the vendor promises a reduction of 25 percent in that time. Annual usage of soap is 4,500 gallons. The car wash is open 360 days a year. Assume that daily usage is normal, and that it has a standard deviation of 2 gallons per day. The ordering cost is $30 and annual carrying cost is $3 a gallon. The revised price list (cost per gallon) is shown in the following table:

Quantity	Unit Price
1–399	$2.00
400–799	1.70
800+	1.62

a. What order quantity is optimal?
b. What ROP is appropriate if the acceptable risk of a stockout is 1.5 percent?

26. A small copy center uses five 500-sheet boxes of copy paper a week. Experience suggests that usage can be well approximated by a normal distribution with a mean of five boxes per week and a standard deviation of one-half box per week. Two weeks are required to fill an order for letterhead stationery. Ordering cost is $2, and annual holding cost is 20 cents per box.

a. Determine the economic order quantity, assuming a 52-week year.
b. If the copy center reorders when the supply on hand is 12 boxes, compute the risk of a stockout.
c. If a fixed interval of seven weeks instead of an ROP is used for reordering, what risk does the copy center incur that it will run out of stationery before this order arrives if it orders 36 boxes when the amount on hand is 12 boxes?

27. Ned's Natural Foods sells unshelled peanuts by the pound. Historically, Ned has observed that daily demand is normally distributed with a mean of 80 pounds and a standard deviation of 10 pounds. Lead time also appears normally distributed with a mean of eight days and a standard deviation of one day.

a. What ROP would provide a stockout risk of 10 percent during lead time?
b. What is the expected number of units (pounds) short per cycle?

28. Regional Supermarket is open 360 days per year. Daily use of cash register tape averages 10 rolls. Usage appears normally distributed with a standard deviation of 2 rolls per day. The cost of ordering tape is $1, and carrying costs are 40 cents per roll a year. Lead time is three days.

a. What is the EOQ?
b. What ROP will provide a lead time service level of 96 percent?
c. What is the expected number of units short per cycle with 96 percent? Per year?
d. What is the annual service level?

29. A service station uses 1,200 cases of oil a year. Ordering cost is $40, and annual carrying cost is $3 per case. The station owner has specified an *annual* service level of 99 percent.

a. What level of safety stock is appropriate if lead time demand is normally distributed with a mean of 80 cases and a standard deviation of 5 cases?
b. What is the risk of a stockout during lead time?

30. Weekly demand for diesel fuel at a department of parks depot is 250 gallons. The depot operates 52 weeks a year. Weekly usage is normal and has a standard deviation of 14 gallons. Holding cost for the fuel is $1 a month, and it costs $20 in administrative time to submit an order for more fuel. It takes one-half week to receive a delivery of diesel fuel. Determine the amount of safety stock that would be needed if the manager wants

a. An annual service level of 98 percent. What is the implication of negative safety stock?
b. The expected number of units short per order cycle to be no more than 5 gallons.

31. A drugstore uses fixed-order cycles for many of the items it stocks. The manager wants a service level of .98. The order interval is 14 days, and lead time is 2 days. Average demand for one item is 40 units per day, and the standard deviation of demand is 3 units per day. Given the on-hand inventory at the reorder time for each order cycle shown in the following table, determine the order quantities for cycles 2, 3, and 4:

Cycle	On Hand
1	42
2	8
3	103

32. A manager must set up inventory ordering systems for two new production items, P34 and P35. P34 can be ordered at any time, but P35 can be ordered only once every four weeks. The company operates 50 weeks a year, and the weekly usage rates for both items are normally distributed. The manager has gathered the following information about the items:

	Item P34	Item P35
Average weekly demand	60 units	70 units
Standard deviation	4 units per week	5 units per week
Unit cost	$15	$20
Annual holding cost	30%	30%
Ordering cost	$70	$30
Lead time	2 weeks	2 weeks
Acceptable stockout risk	2.5%	2.5%

a. When should the manager reorder each item?
b. Compute the order quantity for P34.
c. Compute the order quantity for P35 if 110 units are on hand at the time the order is placed.

33. Given the following list of items,
a. Classify the items as A, B, or C.
b. Determine the economic order quantity for each item (round to the nearest whole unit).

Item	Estimated Annual Demand	Ordering Cost	Holding Cost (%)	Unit Price
H4-010	20,000	50	20	2.50
H5-201	60,200	60	20	4.00
P6-400	9,800	80	30	28.50
P6-401	14,500	50	30	12.00
P7-100	6,250	50	30	9.00
P9-103	7,500	50	40	22.00
TS-300	21,000	40	25	45.00
TS-400	45,000	40	25	40.00
TS-041	800	40	25	20.00
V1-001	33,100	25	35	4.00

34. Demand for jelly doughnuts on Saturdays at Don's Doughnut Shoppe is shown in the following table. Determine the optimal number of doughnuts, in dozens, to stock if labor, materials, and overhead are estimated to be $3.20 per dozen, doughnuts are sold for $4.80 per dozen, and left-over doughnuts at the end of each day are sold the next day at half price. What is the *resulting* service level?

Demand (dozens)	Relative Frequency
19	.01
20	.05
21	.12
22	.18
23	.13
24	.14
25	.10
26	.11
27	.10
28	.04
29	.02

35. A public utility intends to buy a turbine as part of an expansion plan and must now decide on the number of spare parts to order. One part, no. X135, can be purchased for $100 each. Carrying and disposal costs are estimated to be 145 percent of the purchase price over the life of the turbine. A stockout would cost roughly $88,000 due to downtime, ordering, and "special purchase" factors. Historical records based on the performance of similar equipment operating under similar conditions suggest that demand for spare parts will tend to approximate a Poisson distribution with a mean of 3.2 parts for the useful life of the turbine.

a. What is the optimal number of spares to order?

b. Carrying no spare parts would be the best strategy for what range of shortage cost?

36. Skinner's Fish Market buys fresh Boston bluefish daily for $4.20 per pound and sells it for $5.70 per pound. At the end of each business day, any remaining bluefish is sold to a producer of cat food for $2.40 per pound. Daily demand can be approximated by a normal distribution with a mean of 80 pounds and a standard deviation of 10 pounds. What is the optimal stocking level?

37. A small grocery store sells fresh produce, which it obtains from a local farmer. During the strawberry season, demand for fresh strawberries can be reasonably approximated using a normal distribution with a mean of 40 quarts per day and a standard deviation of 6 quarts per day. Excess costs run 35 cents per quart. The grocer orders 49 quarts per day.

a. What is the implied cost of shortage per quart?

b. Why might this be a reasonable figure?

38. Demand for devil's food whipped-cream layer cake at a local pastry shop can be approximated using a Poisson distribution with a mean of six per day. The manager estimates it costs $9 to prepare each cake. Fresh cakes sell for $12. Day-old cakes sell for $9 each. What stocking level is appropriate if one-half of the day-old cakes are sold and the rest thrown out?

39. Burger Prince buys top-grade ground beef for $1.00 per pound. A large sign over the entrance guarantees that the meat is fresh daily. Any leftover meat is sold to the local high school cafeteria for 80 cents per pound. Four hamburgers can be prepared from each pound of meat. Burgers sell for 60 cents each. Labor, overhead, meat, buns, and condiments cost 50 cents per burger. Demand is normally distributed with a mean of 400 pounds per day and a standard deviation of 50 pounds per day. What daily order quantity is optimal? (Hint: Shortage cost must be in dollars per pound.)

40. Demand for rug-cleaning machines at Clyde's U-Rent-It is shown in the following table. Machines are rented by the day only. Profit on the rug cleaners is $10 per day. Clyde has four rug-cleaning machines.

Demand	Frequency
0	.30
1	.20
2	.20
3	.15
4	.10
5	.05
	1.00

a. Assuming that Clyde's stocking decision is optimal, what is the implied range of excess cost per machine?

b. Your answer from part *a* has been presented to Clyde, who protests that the amount is too low. Does this suggest an increase or a decrease in the number of rug machines he stocks? Explain.

c. Suppose now that the $10 mentioned as profit is instead the excess cost per day for each machine and that the shortage cost is unknown. Assuming that the optimal number of machines is four, what is the implied range of shortage cost per machine?

41. A manager is going to purchase new processing equipment and must decide on the number of spare parts to order with the new equipment. The spares cost $200 each, and any unused spares will have an expected salvage value of $50 each. The probability of usage can be described by this distribution:

Number	0	1	2	3
Probability	.10	.50	.25	.15

If a part fails and a spare is not available, it will take two days to obtain a replacement and install it. The cost for idle equipment is $500 per day. What quantity of spares should be ordered?

42. A Las Vegas supermarket bakery must decide how many wedding cakes to prepare for the upcoming weekend. Cakes cost $33 each to make, and they sell for $60 each. Unsold cakes are reduced to half-price on Monday, and typically one-third of those are sold. Any that remain are donated to a nearby senior center. Analysis of recent demand resulted in the following table:

Demand	0	1	2	3
Probability	.15	.35	.30	.20

How many cakes should be prepared to maximize expected profit?

43. Offwego Airlines has a daily flight from Chicago to Las Vegas. On average, 18 ticket holders cancel their reservations, so the company intentionally overbooks the flight. Cancellations can be described by a normal distribution with a mean of 18 passengers and a standard deviation of 4.55 passengers. Profit per passenger is $99. If a passenger arrives but cannot board due to overbooking, the company policy is to provide a cash payment of $200. How many tickets should be overbooked to maximize expected profit?

44. Caring Hospital's dispensary reorders doses of a drug when the supply on hand falls to 18 units. Lead time for resupply is three days. Given the typical usage over the last 10 days, what service level is achieved with the hospital's reorder policy? Hint: Use Formula 12–13.

Day	1	2	3	4	5	6	7	8	9	10
Units	3	4	7	5	5	6	4	3	4	5

UPD Manufacturing

CASE

UPD Manufacturing produces a range of health care appliances for hospital as well as for home use. The company has experienced a steady demand for its products, which are highly regarded in the health care field. Recently the company has undertaken a review of its inventory ordering procedures as part of a larger effort to reduce costs.

One of the company's products is a blood pressure testing kit. UPD manufactures all of the components for the kit in-house except for the digital display unit. The display units are ordered at six-week intervals from the supplier. This ordering system began about five years ago, because the supplier insisted on it. However, that supplier was bought out by another supplier about a year ago, and the six-week ordering requirement is no longer in place. Nonetheless, UPD has continued to use the six-week ordering policy. According to purchasing manager Tom Chambers, "Unless somebody can give me a reason for changing, I'm going to stick with what we've been doing. I don't have time to reinvent the wheel."

Further discussions with Tom revealed a cost of $32 to order and receive a shipment of display units from the supplier. The company assembles 89 kits a week. Also, information from Sara James, in Accounting, indicated a weekly carrying cost of $.08 for each display unit.

The supplier has been quite reliable with deliveries; orders are received five working days after they are faxed to the supplier. Tom indicated that as far as he was concerned, lead-time variability is virtually nonexistent.

Questions

1. Would using an order interval other than every six weeks reduce costs? If so, what order interval would be best, and what order size would that involve?
2. Would you recommend changing to the optimal order interval? Explain.

Harvey Industries

CASE

Background

Harvey Industries, a Wisconsin company, specializes in the assembly of high-pressure washer systems and in the sale of repair parts for these systems. The products range from small portable high-pressure washers to large industrial installations for snow removal from vehicles stored outdoors during the winter months. Typical uses for high-pressure water cleaning include:

Automobiles

Airplanes

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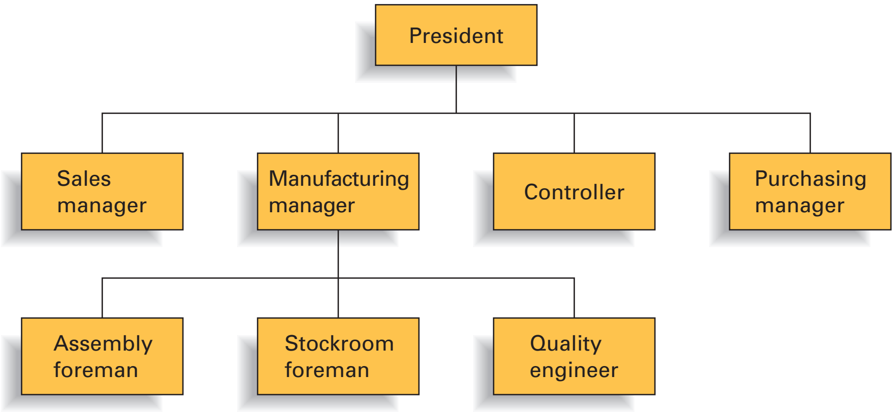

Building maintenance

Barns

Engines

Ice cream plants

Lift trucks

Machinery

Swimming pools

Industrial customers include General Motors, Ford, Chrysler, Delta Airlines, United Parcel Service, and Shell Oil Company.

Although the industrial applications are a significant part of its sales, Harvey Industries is primarily an assembler of equipment for coin operated self-service car wash systems. The typical car wash is of concrete block construction with an equipment room in the center, flanked on either side by a number of bays. The cars are driven into the bays where the owner can wash and wax the car, utilizing high-pressure hot water and liquid wax. A dollar bill changer is available to provide change for the use of the equipment and the purchase of various products from dispensers. The products include towels, tire cleaner, and upholstery cleaner.

In recent years Harvey Industries has been in financial difficulty. The company has lost money for three of the last four years, with the last year's loss being $17,174 on sales of $1,238,674. Inventory levels have been steadily increasing to their present levels of $124,324.

The company employs 23 people with the management team consisting of the following key employees: president, sales manager, manufacturing manager, controller, and purchasing manager. The abbreviated organization chart reflects the reporting relationship of the key employees and the three individuals who report directly to the manufacturing manager.

Current Inventory Control System

The current inventory control "system" consists of orders for stock replenishment being made by the stockroom foreman, the purchasing manager, or the manufacturing manager whenever one of them notices that the inventory is low. An order for replenishment of inventory is also placed whenever someone (either a customer or an employee in the assembly area) wants an item and it is not in stock.

Some inventory is needed for the assembly of the high-pressure equipment for the car wash and industrial applications. There are current and accurate bills of material for these assemblies. The material needs to support the assembly schedule are generally known well in advance of the build schedule.

The majority of inventory transactions are for repair parts and for supplies used by the car washes, such as paper towels, detergent, and wax concentrate. Because of the constant and rugged use of the car wash equipment, there is a steady demand for the various repair parts.

The stockroom is well organized, with parts stored in locations according to each vendor. The number of vendors is relatively limited, with each vendor generally supplying many different parts. For example, the repair parts from Allen Bradley, a manufacturer of electrical motors, are stocked in the same location. These repair parts will be used to provide service for the many electrical motors that are part of the high-pressure pump and motor assembly used by all of the car washes.

Because of the heavy sales volume of repair parts, there are generally two employees working in the stockroom—a stockroom foreman who reports to the manufacturing manager and an assistant to the foreman. One of these two employees will handle customer orders. Many customers stop by and order the parts and supplies they need. Telephone orders are also received and are shipped by United Parcel Service the same day.

The assembly area has some inventory stored on the shop floor. This inventory consists of low-value items that are used every day, such as nuts, bolts, screws, and washers. These purchased items do not amount to very much dollar volume throughout the year. Unfortunately, oftentimes the assembly area is out of one of these basic items and this causes a significant amount of downtime for the assembly lines.

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Paperwork is kept to a minimum. A sales slip listing the part numbers and quantities sold to a customer is generally made out for each sale. If the assembly department needs items that are not stocked on the assembly floor, someone from that department will enter the stockroom and withdraw the necessary material. There is no paperwork made out for the items needed on the assembly floor.

There were 973 different part numbers purchased for stock last year and those purchases amounted to $314,673. An analysis of inventory records shows that $220,684 was spent on just 179 of the part numbers.

Fortunately for Harvey Industries, most of the items they purchase are stocked by either the manufacturer or by a wholesaler. When it is discovered that the company is out of stock on an item, it generally takes only two or three days to replenish the stock.

Due to the company's recent losses, its auditing firm became concerned about the company's ability to continue in business. Recently the company sold off excess vacant land adjoining its manufacturing facility to generate cash to meet its financial obligations.

New President

Because of the recent death of the owner, the trust department of a Milwaukee Bank (as trustee for the state) has taken over the company's affairs and has appointed a new company president. The new president has identified many problem areas—one of which is improper inventory control. He has retained you as a consultant to make specific recommendations concerning a revised inventory control system. What are your recommendations and their rationale?

Grill Rite

CASE

Grill Rite is an old-line company that started out making wooden matches. As that business waned, the company entered the electric barbecue grill market, with five models of grills it sells nationally. For many years the company maintained a single warehouse from which it supplied its distributors.

The plant where the company produces barbecue sets is located in a small town, and many workers have been with the company for many years. During the transition from wooden matches to barbecue grills, many employees gave up their weekends to help with changing over the plant and learning the new skills they would need, without pay. In fact, Mac Wilson, the company president, can reel off a string of such instances of worker loyalty. He has vowed to never layoff any workers, and to maintain a full employment, steady rate of output. "Yes, I know demand for these babies (barbecue grills) is seasonal, but the inventory boys will just have to deal with it. On an annual basis, our output matches sales."

Inventory is handled by a system of four warehouses. There is a central warehouse located near the plant that supplies some customers directly, and the three regional warehouses.

The vice president for sales, Julie Berry, is becoming increasingly frustrated with the inventory system that she says "is antiquated and unresponsive." She points to increasing complaints from regional sales managers about poor customer service, saying customer orders go unfilled or are late, apparently due to shortages at the regional warehouse. Regional warehouse managers, stung by complaints from sales managers, have responded by increasing their order sizes from the main warehouse, and maintaining larger amounts of safety stock. This has resulted in increased inventory holding costs, but it hasn't eliminated the problem. Complaints are still coming in from sales people about shortages and lost sales. According to managers of the regional warehouses, their orders to the main warehouse aren't being shipped, or when they are, they are smaller quantities than requested. The manager of the main warehouse, Jimmy Joe ("JJ") Sorely, says his policy is to give preference to "filling direct orders from actual customers, rather than warehouse orders that might simply reflect warehouses trying to replenish their safety stock. And besides, I never know when I'll get hit with an order from one of the regional warehouses. I guess they think we've got an unlimited supply." Then he adds, "I thought when we added the warehouses, we could just divide our inventory among the warehouses, and everything would be okay."

When informed of the "actual customers" remark, a regional warehouse manager exclaimed, "We're their biggest customer!"

Julie Berry also mentioned that on more than one occasion she has found that items that were out of stock at one regional warehouse were in ample supply in at least one other regional warehouse.

Take the position of a consultant called in by president Mac Wilson. What recommendations can you make to alleviate the problems the company is encountering?

Bruegger's Bagel Bakery

OPERATIONS TOUR

www.brueggers.com

Bruegger's Bagel Bakery makes and sells a variety of bagels, including plain, onion, poppyseed, and cinnamon raisin, as well as assorted flavors of cream cheese. Bagels are the major source of revenue for the company.

The bagel business is a $3 billion industry. Bagels are very popular with consumers. Not only are they relatively low in fat, they are filling, and they taste good! Investors like the bagel industry because it can be highly profitable: it only costs about $.10 to make a bagel, and they can be sold for $.50 each or more. Although some bagel companies have done poorly in recent years, due mainly to poor management, Bruegger's business is booming; it is number one nationally, with over 450 shops that sell bagels, coffee, and bagel sandwiches for takeout or on-premise consumption. Many stores in the Bruegger's chain generate an average of $800,000 in sales annually.

Production of bagels is done in batches, according to flavor, with each flavor being produced on a daily basis. Production of bagels at Bruegger's begins at a processing plant, where the basic ingredients of flour, water, yeast, and flavorings are combined in a special mixing machine. After the dough has been thoroughly mixed, it is transferred to another machine that shapes the dough into individual bagels. Once the bagels have been formed, they are loaded onto refrigerated trucks for shipping to individual stores. When the bagels reach a store, they are unloaded from the trucks and temporarily stored while they rise. The final two steps of processing involve boiling the bagels in a kettle of water and malt for one minute, and then baking the bagels in an oven for approximately 15 minutes.

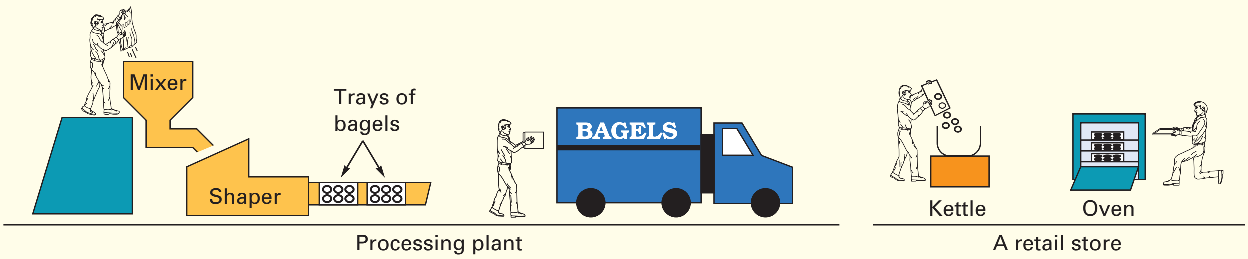
The process is depicted in the Figure.

Quality is an important feature of a successful business. Customers judge the quality of bagels by their appearance (size, shape, and shine), taste, and consistency. Customers are also sensitive to the service they receive when they make their purchases. Bruegger's devotes careful attention to quality at every stage of operation, from choosing suppliers of ingredients, careful monitoring of ingredients, and keeping equipment in good operating condition to monitoring output at each step in the process. At the stores, employees are instructed to watch for deformed bagels and to remove them when they find them. (Deformed bagels are returned to a processing plant where they are sliced into bagel chips, packaged, and then taken back to the stores for sale, thereby reducing the scrap rate.) Employees who work in the stores are carefully chosen and then trained so that they are competent to operate the necessary equipment in the stores and to provide the desired level of service to customers.

The company operates with minimal inventories of raw materials and inventories of partially completed bagels at the plant and very little inventory of bagels at the stores. One reason for this is to maintain a high degree of freshness in the final product by continually supplying fresh product to the stores. A second reason is to keep costs down; minimal inventories mean less space is needed for storage.

Questions

1. Bruegger's maintains relatively little inventory at either its plants or its retail stores. List the benefits and risks of this policy.
2. Quality is very important to Bruegger's.
   a. What features of bagels do customers look at to judge their quality?
   b. At what points in the production process do workers check bagel quality?
   c. List the steps in the production process, beginning with purchasing ingredients, and ending with the sale, and state how quality can be positively affected at each step.
3. Which inventory models could be used for ordering the ingredients for bagels? Which model do you think would be most appropriate for deciding how many bagels to make in a given batch?
4. Bruegger's has bagel-making machines at its plants. Another possibility would be to have a bagel-making machine at each store. What advantages does each alternative have?



PSC, Inc.

OPERATIONS TOUR

www.pscnet.com

PSC designs and produces a variety of laser bar code scanning devices. The products include handheld bar code readers, high-speed fixed-position industrial scanners, and retail checkout scanners as well as a full line of accessories, software, and supplies to support its products. Headquartered in Eugene, Oregon, the company has manufacturing facilities in Eugene, and Paris, France, with roughly 1,200 employees worldwide.

Products

Bar code scanners are designed for a variety of situations that can involve long range scanning, reading small bar codes, and performing high-speed scans. They are used extensively in industry, business, and government to manage and control the entire supply chain, which includes suppliers, production, warehousing, distribution, retail sales, and service. Examples of bar code readers include the familiar point-of-sale scanners encountered at supermarkets and other retail stores. They come in a variety of forms, ranging from handheld to built-in models. High-speed, unattended scanners are used for automated material handling and sorting. Typical installations include high-volume distribution centers such as JC Penney's catalog operation and airport baggage handling systems. The company also produces "reader engines" that it supplies to other companies for use in their products. These may be as small as 1.2 cubic inches. One application for an "engine product" is found in lottery ticket validation machines. Use of bar code readers has greatly increased the speed and accuracy of data collection, resulting in increased productivity, improved production and inventory tracking and control, and improved market information.

Operations

Forecasting

Forecasting is not a significant activity at PSC due to several factors. There is high standardization of scanner components, which creates stability in usage requirements. Supplier lead times are relatively short, often only a few days. Orders are typically small; 70 percent of all orders are for 10 units or less. There is a fair degree of production flexibility, particularly in terms of product customization. As a result of these factors, the company relies mainly on short-term, moving average forecasts.

Product Design

PSC has developed a robust design in many of its products, enabling them to perform effectively under a broad range of operating conditions. For example, many of its handheld scanners can operate at temperatures ranging from −22° F to 120° F, and can withstand drops onto concrete surfaces from heights up to six feet and still function. This has enabled the company to offer warranties ranging from 24 to 36 months, far exceeding the industry standard of 3 to 12 months.

Layout

PSC has developed an efficient production layout that consists of assembly lines and work centers. The assembly lines handle standardized production and subassemblies and the work centers handle final assembly and customization of products. Assembly lines are U-shaped to facilitate communication among workers. The work centers are designed for production flexibility; they can be reconfigured in about four hours. Work centers are staffed by teams of three to six cross-trained workers who are responsible for an order from start to finish.

The Production Process

Production involves a combination of assembly line and batch processing that provides high volume and flexibility to customized individual orders. Because of the high standardization among the internal components of different scanners, many of the subassemblies can be produced on assembly lines. Customization is done primarily on the external portion of various products according to customer specification.

The production process for scanner engines is depicted in the process flowchart shown in the figure. The process begins when an order is received from a customer. The order is then configured according to customer specifications. Next it is entered into the computer to obtain a bill of materials (BOM), and the order is transmitted to production control so that it can be scheduled for production. A "traveler" packet containing product specifications and the BOM is created. It will accompany the order throughout the process.

The traveler is sent to the "kitting" area where standard parts and any customized parts are obtained and placed into a bin ("kit") and then placed in a flow rack until the assigned work center is ready for the job (i.e., a pull system).

The next phase of the process transforms unprogrammed, panelized circuit boards into programmed boards. The boards first pass through a screen printer which uses a stencil to coat the boards with a solder paste. Next the boards pass through a chip mounter which enters values for the smaller, passive components of the circuit board at a rate of 25,000 parts per hour. A second mounter enters values for the larger, programmable components at a rate of 7,000 parts per hour. The slower rate for the larger components is offset by the fact that there are fewer of those components. The process ends up being balanced, and no bottlenecks occur.

The programmed boards move by conveyor to a station for visual inspection. Rejects are returned to the chip mounter area, and boards that pass are sent through an oven to solidify the solder, making the programming permanent. The circuit boards are then removed from the panels and placed into the kit. The kits are then taken to designated work centers for customization and placement in scanner engines.

Work centers typically have builders, computer operators, and a tester. A builder mounts the laser diodes on the circuit board and passes it to a computer operator who downloads the customer

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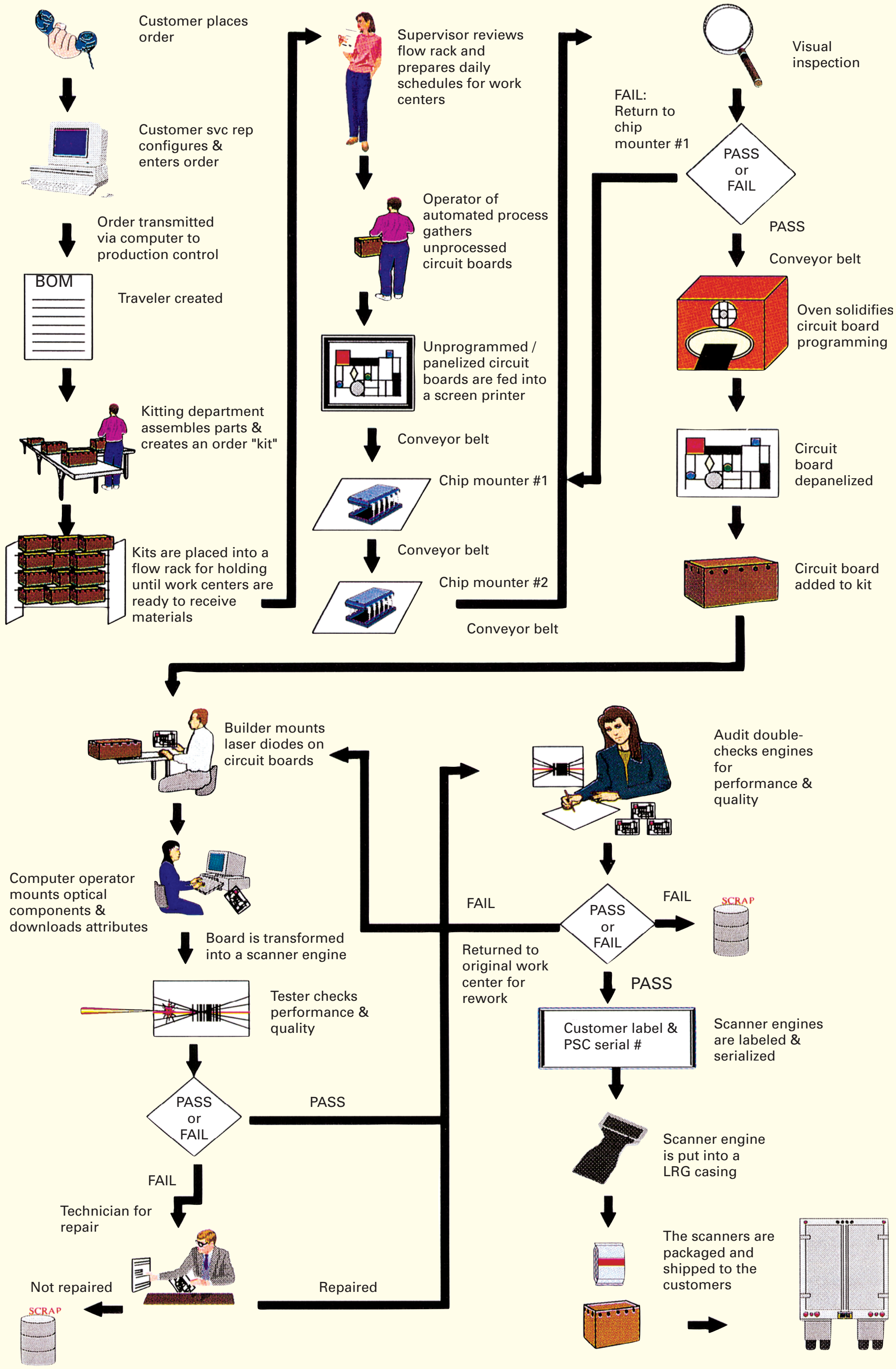
PSC Inc. Scanner Engine Production Process Flowchart



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specifications into the microprocessor of the scan engine. The operator also mounts the optical components and adjusts them for the design of the scanner (e.g., long range scanning). Next, the engine goes to the tester, who checks to make sure that the scanner is capable of reading bar codes and laser characteristics. Engines that fail are sent for repair and later retested. If the engine fails a second time, it is either returned for further repair or scrapped. Engines which pass are placed in an electrostatic bag which protects them from static electricity that could damage the programming.

Engines are then sent to Audit for another check for performance quality.

Engines that pass are incorporated into the final product, a serial number is added, along with a label, and the product is sent to the packing area and then shipped to the customer.

Inventory The company uses a variety of methods for inventory management, and it attempts to minimize the amount of inventory. A computer determines component requirements and generates purchase orders for the components for each order, and then appropriate orders for various components from vendors are prepared. However, the company maintains a stock of standard components that are replenished using a reorderpoint system. The company has adopted point-of-use replenishment for some areas of operations, having deliveries come directly to the production floor. Finished products are immediately shipped to the customer, which enhances the company's delivery performance and avoids finished goods inventory.

Suppliers Approximately 40 vendors supply parts and materials to PSC, each of which has been subjected to a multiple-step supplier certification program that includes the supplier completing a self-evaluation questionnaire; an on-site visit of supplier facilities by a team from PSC made up of people from engineering, purchasing, and operations; a probation period; and rating of products using government MIL-STD 105 specifications. Vendor performance is tracked on product quality, delivery, and service.

When an item is removed from inventory, it is scanned into the computer, and this information is transmitted directly to suppliers, along with purchase orders to restock components.

Quality Quality is strongly emphasized at PSC. Employees are trained in quality concepts and the use of quality tools. Training is incorporated on-the-job so that employees can see the practical applications of what they are learning. Employees are responsible for performing in-process quality checks (quality at the source), and to report any defects they discover to their supervisor. Defects are assigned to one of three categories for problem solving:

- Operator/training error. The supervisor notifies a trainer who then provides appropriate retraining.
- Process/equipment problem. The supervisor notifies the manufacturing engineer who is then responsible for diagnosing the cause and correcting the problem.
- Parts/material problem. The supervisor notifies quality assurance, who then notifies the vendor to correct the problem. Defective parts are either scrapped or returned to the vendor.

Lean Production

PSC strives to operate on lean production principles. In addition to emphasizing high levels of quality, production flexibility, low levels of inventories, and having some deliveries come right to the production floor, its organization structure is fairly flat, and it uses a team approach. Still another feature of lean production is that many of PSC's workers are multiskilled. The company encourages employees to master new skills through a pay-for-skill program, and bases hourly pay rates on the number of skills a worker can perform.

Business Strategy

The company has developed what it believes is a strong strategy for success. Strategic initiatives include anticipating customer demand for miniaturization and the ability to customize products; expanding its proprietary technology; and expanding internationally into Western Europe (now accounts for about 35 percent of sales) and the Pacific rim (now accounts for about 10 percent of sales). Several plants or groups are ISO certified, which has been important for European sales. The company intends to continue to expand its product lines through acquisition of other companies.

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