Business Statistics: A Decision-Making Approach

6th Edition

Chapter 8

Introduction to Hypothesis Testing

Chapter Goals

After completing this chapter, you should be able to:

- Formulate null and alternative hypotheses for applications involving a single population mean or proportion
- Formulate a decision rule for testing a hypothesis
- Know how to use the test statistic, critical value, and p-value approaches to test the null hypothesis

Testing Theories

Hypotheses Competing theories that we want to test about a population are called *Hypotheses* in statistics. Specifically, we label these competing theories as *Null Hypothesis* (H_0) and *Alternative Hypothesis* (H_1 or H_A).

H_0 **:** The null hypothesis is the status quo or the prevailing viewpoint.

H_A **:** The alternative hypothesis is the competing belief. It is the statement that the researcher is hoping to prove.

The Null Hypothesis, H_0

(continued)

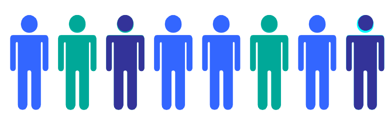
- Begin with the assumption that the null hypothesis is true
- Refers to the status quo
- Always contains "=" , "≤" or "≥" sign
- May or may not be rejected

The Alternative Hypothesis, H_A

- Challenges the status quo
- Never contains the "=" , "≤" or "≥" sign
- Is generally the hypothesis that is believed (or needs to be supported) by the researcher
- Provides the "direction of extreme"

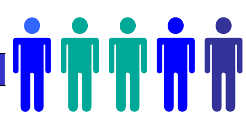
Hypothesis Testing Process

Claim: the population mean age is 50. (Null Hypothesis: H_0: $\mu = 50$)



Population

Now select a random sample



Sample

Suppose the sample mean age is 20: $\bar{x} = 20$

Is $\bar{x} = 20$ likely if $\mu = 50$?

If not likely, REJECT Null Hypothesis

Deciding Which Theory to Support

Decision making is based on the “rare event” concept. Since the null hypothesis is the status quo, we assume that it is true unless the observed result is extremely unlikely (rare) under the null hypothesis.

- **Definition**: *If the data were indeed unlikely to be observed under the assumption that H_0 is true, and therefore we reject H_0 in favor of H_A, then we say that the data are **statistically significant**.*

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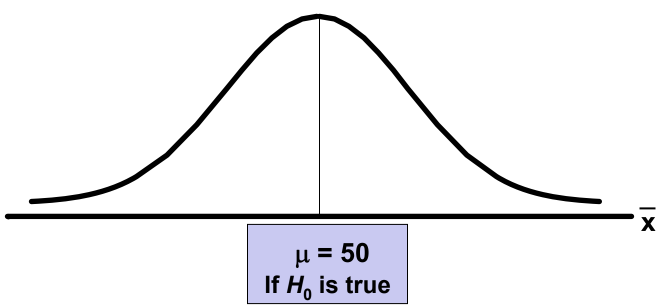
Reason for Rejecting H_0

Sampling Distribution of $\bar{x}$



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Level of Significance, α

- **Defines unlikely values of sample statistic if null hypothesis is true**
  - Defines rejection region of the sampling distribution
- Is designated by α , (level of significance)
- Is selected by the researcher at the beginning
- Provides the critical value(s) of the test

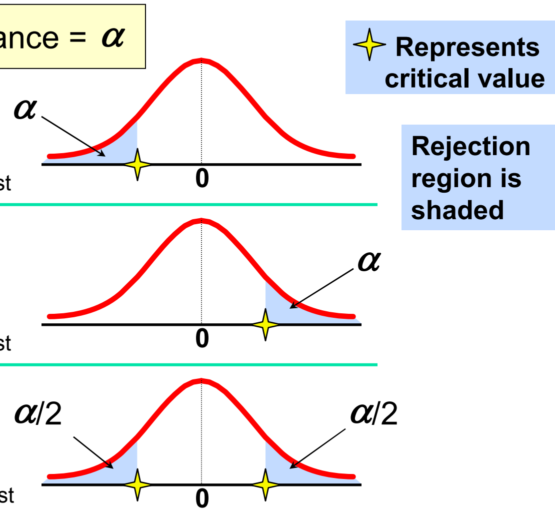
Level of Significance and the Rejection Region

Level of significance = α

H_0: μ ≥ 3
H_A: μ < 3
Lower tail test

H_0: μ ≤ 3
H_A: μ > 3
Upper tail test

H_0: μ = 3
H_A: μ ≠ 3
Two tailed test



Critical Value Approach to Testing

- Convert sample statistic (e.g.: $\bar{x}$) to test statistic (*Z** *or* *t** statistic)
- Determine the critical value(s) for a specified level of significance α from a table or computer
- If the test statistic falls in the rejection region, reject H_0 ; otherwise do not reject H_0

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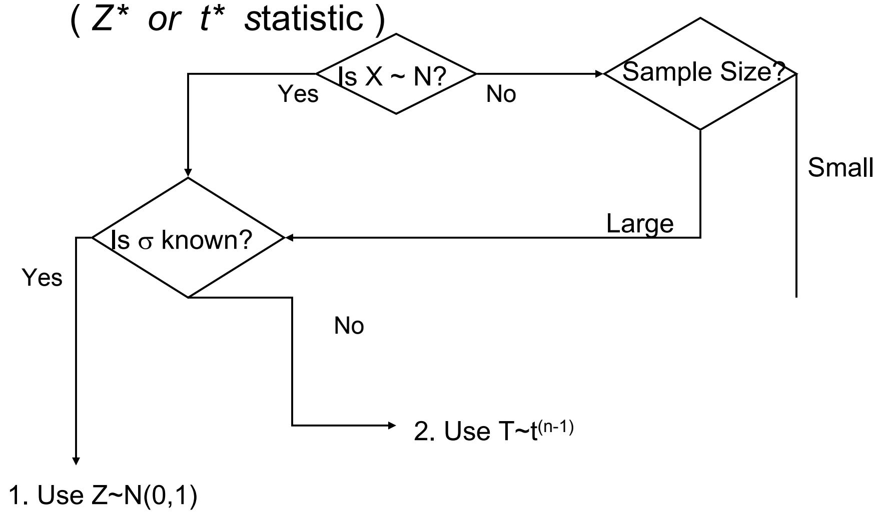
Critical Value Approach to Testing

- Convert sample statistic ($\bar{x}$) to a test statistic (*Z** *or* *t** statistic)



Calculating the Test Statistic

Z Test Statistic

- **Two-Sided: $H_0 : \mu = \mu_0$; $H_A : \mu \neq \mu_0$**
  - Reject H_0 if $Z^* > Z_{(1-\alpha/2)}$ or $Z^* < -Z_{(1-\alpha/2)}$, otherwise do not reject H_0
- **One-Sided Upper Tail: $H_0 : \mu \leq \mu_0$; $H_A : \mu > \mu_0$**
  - Reject H_0 if $Z^* > Z_{(1-\alpha)}$, otherwise do not reject H_0
- **One-Sided Lower Tail: $H_0 : \mu \geq \mu_0$; $H_A : \mu < \mu_0$**
  - Reject H_0 if $Z^* < -Z_{(1-\alpha)}$, otherwise do not reject H_0

T test Statistic

- **Two-Sided: $H_0 : \mu = \mu_0$; $H_A : \mu \neq \mu_0$**
  - Reject H_0 if $t^* > t^{(n-1)}_{(1-\alpha/2)}$ or $t^* < -t^{(n-1)}_{(1-\alpha/2)}$, otherwise do not reject H_0
- **One-Sided Upper Tail: $H_0 : \mu \leq \mu_0$; $H_A : \mu > \mu_0$**
  - Reject H_0 if $t^* > t^{(n-1)}_{(1-\alpha)}$, otherwise do not reject H_0
- **One-Sided Lower Tail: $H_0 : \mu \geq \mu_0$; $H_A : \mu < \mu_0$**
  - Reject H_0 if $t^* > t^{(n-1)}_{(1-\alpha)}$, otherwise do not reject H_0

$$z^* = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$t^* = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

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Review: Steps in Hypothesis Testing

1. Specify the population value of interest
2. Formulate the appropriate null and alternative hypotheses
3. Specify the desired level of significance
4. Determine the rejection region
5. Obtain sample evidence and compute the test statistic
6. Reach a decision and interpret the result

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Hypothesis Testing Example

Test the claim that the true mean # of TV sets in US homes is at least 3. Assume that $\sigma = 0.8$

1. Specify the population value of interest

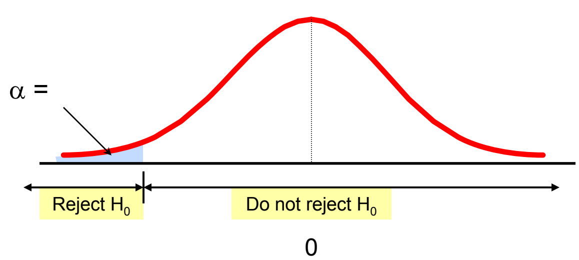
2. Formulate the appropriate null and alternative hypotheses

3. Specify the desired level of significance

Hypothesis Testing Example

(continued)

- 4. Determine the rejection region



Reject H_0 if Z* test statistic <
otherwise do not reject H_0

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Hypothesis Testing Example

- 5. Obtain sample evidence and compute the test statistic

Suppose a sample is taken with the following results: $n = 100$, $\bar{x} = 2.84$ ($\sigma = 0.8$ is assumed known)

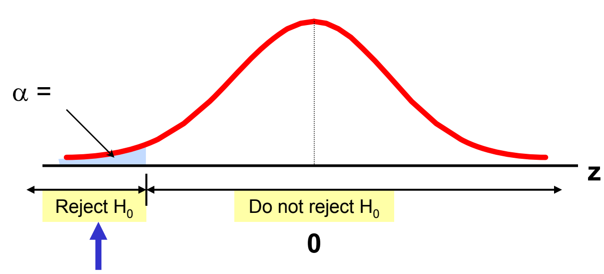
- Then the test statistic is:

$$Z^* = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} =$$

Hypothesis Testing Example

(continued)

- 6. Reach a decision and interpret the result



Since $Z^* = -2.0 <$,

p-Value Approach to Testing

- p-value: Probability of obtaining a test statistic more extreme than the observed sample value given H_0 is true
  - Also called observed level of significance
  - Smallest value of α for which H_0 can be rejected

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p-Value Approach to Testing

- Convert Sample Statistic to Test Statistic (*Z** *or t** statistic)
- Obtain the p-value from a table or computer
- Compare the p-value with α
  - If p-value $< \alpha$, reject H_0
  - If p-value $\geq \alpha$, do not reject H_0

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P-Value Calculation

Z test statistic

- Two-Sided: 2 ×min {P(Z ≥ Z*,Z ≤ Z*)}
- One-Sided Upper Tail P(Z ≥ Z*)
- One-Sided Lower Tail *P*(*Z* ≤ *Z**)

T test statistic

- Two-Sided: 2 ×min {P(t ≥ t*,t ≤ t*)}
- One-Sided Upper Tail P(t ≥ t*)
- One-Sided Lower Tail *P*(*t* ≤ *t**)

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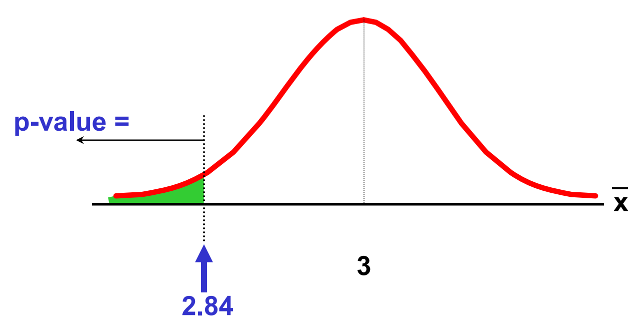
p-value example

- Example: How likely is it to see a sample mean of 2.84 (or something further below the mean) if the true mean is μ = 3.0?



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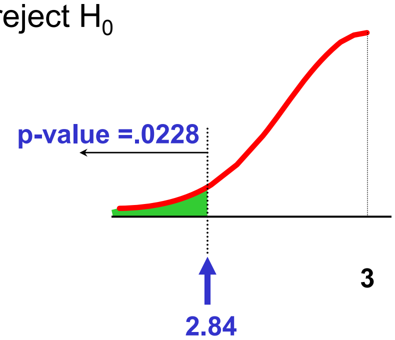
p-value example

(continued)

- Compare the p-value with α
  - If p-value < α , reject H_0
  - If p-value $\geq$ α , do not reject H_0



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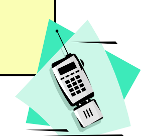
Example: Upper Tail z Test for Mean (σ Known)

A phone industry manager thinks that customer monthly cell phone bill have increased, and now average over $52 per month. The company wishes to test this claim. (Assume σ = 10 is known)



Form hypothesis test:

H_0: $\mu \leq 52$	the average is not over $52 per month
H_A: $\mu > 52$	the average **is** greater than $52 per month (i.e., sufficient evidence exists to support the manager's claim)

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Example: Find Rejection Region

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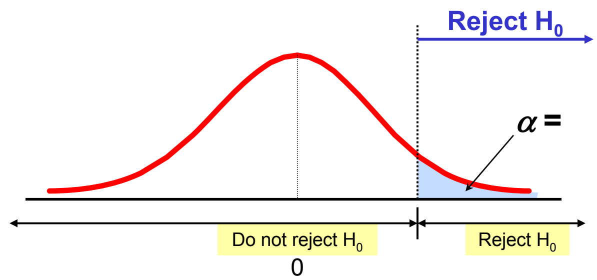


Example: Test Statistic

(continued)

Obtain sample evidence and compute the test statistic

Suppose a sample is taken with the following results: n = 64, $\bar{x}$ = 53.1 (σ=10 was assumed known)

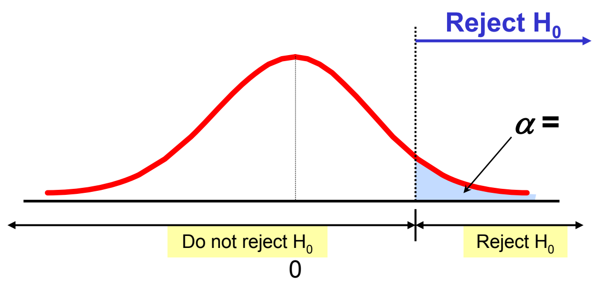
- Then the test statistic is:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} =$$

Example: Decision

(continued)

Reach a decision and interpret the result:



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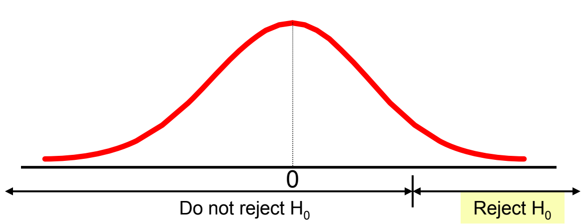
p -Value Solution

(continued)

Calculate the p-value and compare to α



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Example: Two-Tail Test (σ Unknown)

The average cost of a hotel room in New York is said to be $168 per night. A random sample of 25 hotels resulted in $\bar{x}$ = $172.50 and s = $15.40. Test at the α = 0.05 level.

(Assume the population distribution is normal)

H_0: $\mu = 168$
H_A: $\mu \neq 168$

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Outcomes and Probabilities

Possible Hypothesis Test Outcomes

	State of Nature	
Decision	H_0 True	H_0 False
Do Not Reject **H_0**	**No error** **($1 - \alpha$)**	**Type II Error** **(β)**
Reject **H_0**	**Type I Error** **(α)**	**No Error** **($1 - \beta$)**

Key:
Outcome
(Probability)

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