Business Statistics: A Decision-Making Approach

6th Edition

Chapter 7

Estimating Population Values

Confidence Intervals

Content of this chapter

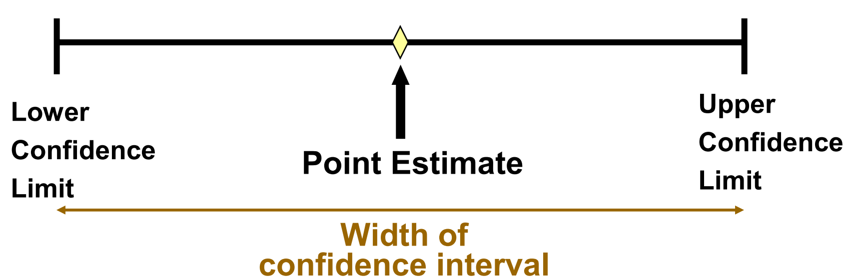
- Confidence Intervals for the Population Mean, μ
  - when Population Standard Deviation σ is Known
  - when Population Standard Deviation σ is Unknown
- Determining the Required Sample Size

Confidence Interval Estimation for μ

- Suppose you are interested in estimating the average amount of money a Kent State Student (population) carries. How would you find out?

Point and Interval Estimates

- A point estimate is a single number,
- a confidence interval provides additional information about variability



Estimation Methods

- Point Estimation
  - Provides single value
  - Based on observations from 1 sample
  - Gives no information on how close value is to the population parameter
- Interval Estimation
  - Provides range of values
  - Based on observations from 1 sample
  - Gives information about closeness to unknown population parameter
    - Stated in terms of “level of confidence.”
    - To determine exactly requires what information?

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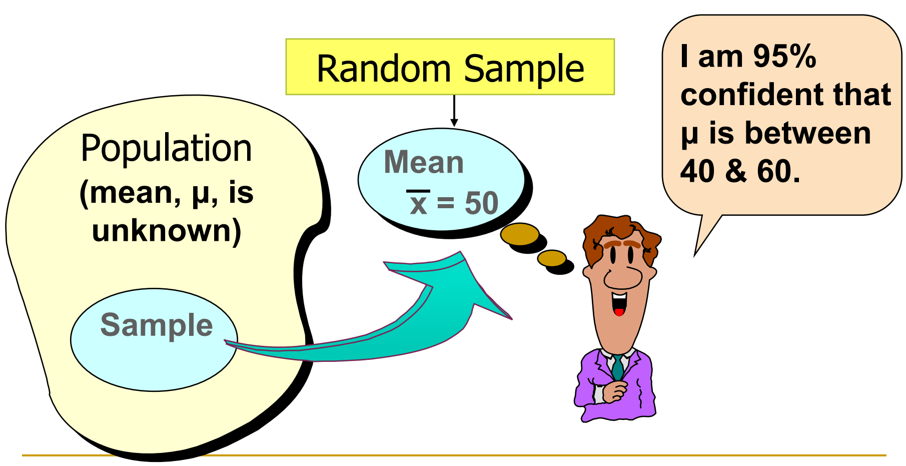
Estimation Process



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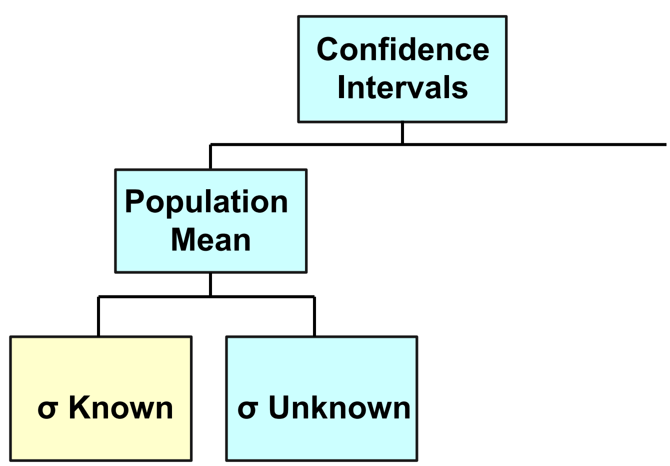
General Formula

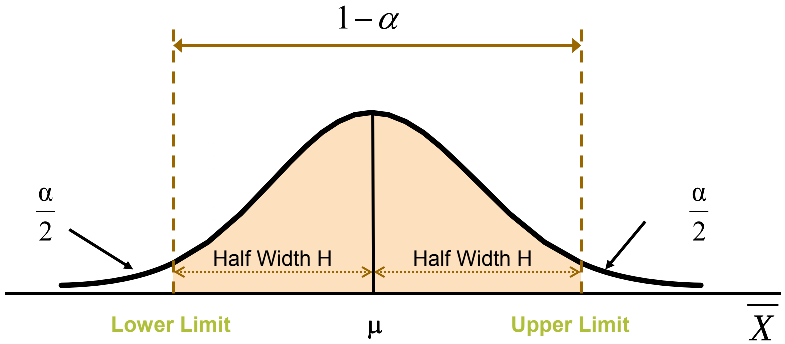
- The general formula for all confidence intervals is:

Point Estimate ± (Critical Value)(Standard Error)

Confidence Intervals



(1-α)x100% Confidence Interval for μ



CI Derivation Continued

1. Parameter = Statistic ± Error (Half Width)

$$\mu = \overline{X} \pm H$$

$$H = \overline{X} - \mu \text{ or } \overline{X} + \mu$$

$$Z = \frac{\overline{X} - \mu}{\sigma_{\overline{X}}} = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}} = \frac{H}{\sigma / \sqrt{n}}$$

$$H = Z \times \sigma / \sqrt{n}$$

$$\mu = \overline{X} \pm Z \times \sigma / \sqrt{n}$$

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Confidence Interval for μ (σ Known)

- Assumptions
  - Population standard deviation σ is known
  - Population is normally distributed
  - If population is not normal, use large sample
- Confidence interval estimate

$$\bar{x} \pm z_{(.5-\alpha/2)} \frac{\sigma}{\sqrt{n}}$$

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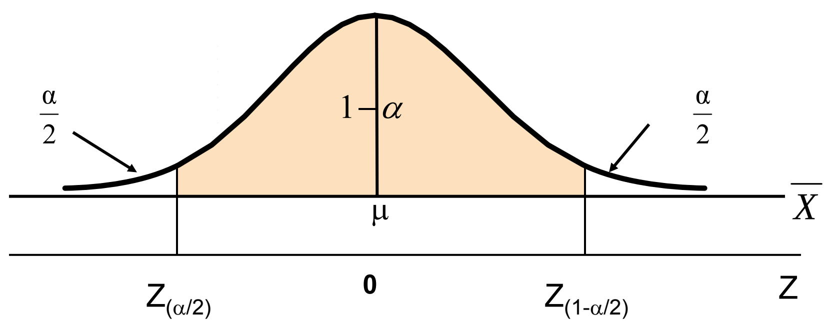
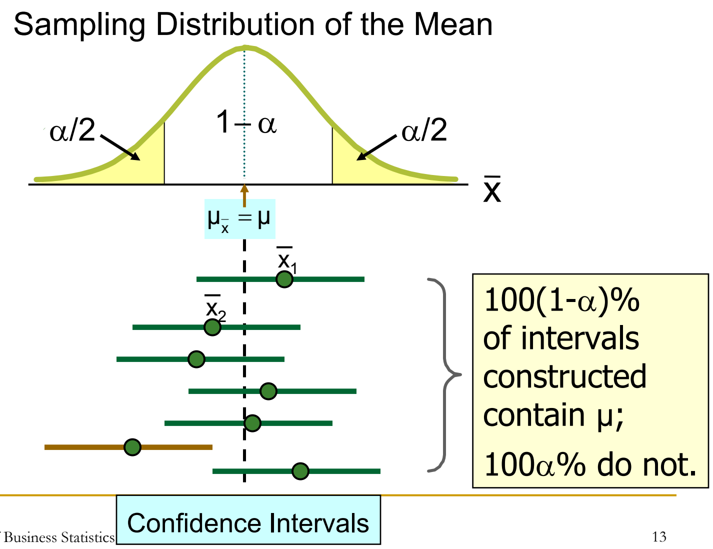
(1-α)x100% CI



Conf. Level	(1-α)	α	(.5-α/2)	$Z_{(.5-\alpha/2)}$
90	0.90	0.10	0.450	
95				
99				

Interpretation



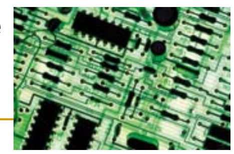
Factors Affecting Half Width

$$H = z_{(.5-\alpha/2)} \frac{\sigma}{\sqrt{n}}$$

- Data variation, σ : H as σ ↓
- Sample size, n : H as n ↑
- Level of confidence, 1 - α : H if 1 - α ↓

Example

- A sample of 11 circuits from a large normal population has a mean resistance of 2.20 ohms. We know from past testing that the population standard deviation is .35 ohms.
- Determine a 95% confidence interval for the true mean resistance of the population.



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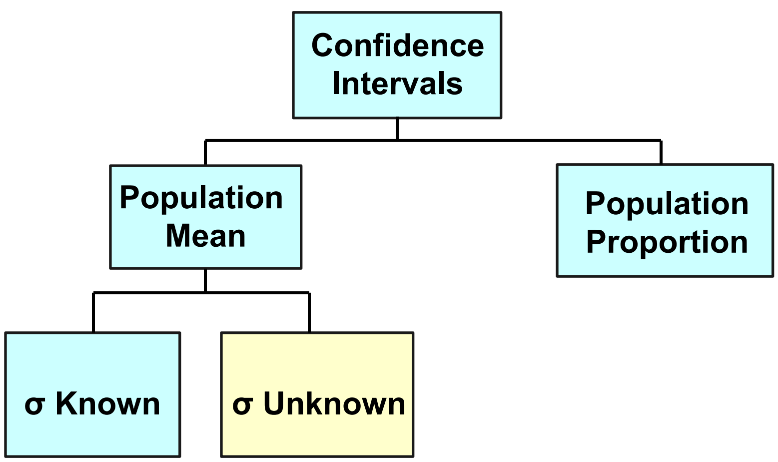
Confidence Intervals



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Confidence Interval for μ (σ Unknown)

- If the population standard deviation σ is unknown, we can substitute the sample standard deviation, s
- This introduces extra uncertainty, since s is variable from sample to sample
- So we use the t distribution instead of the standard normal distribution

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Confidence Interval for μ (σ Unknown)

(continued)

- Assumptions
  - Population standard deviation is unknown
  - Population is normally distributed
  - If population is not normal, use large sample
- Use Student's *t* Distribution
- Confidence Interval Estimate

$$\overline{X} \pm t_{(1-\alpha/2)}^{(n-1)} \frac{s}{\sqrt{n}}$$

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Student's t Distribution

- The t is a family of distributions
- The t value depends on degrees of freedom (d.f.)
  - Number of observations that are free to vary after sample mean has been calculated

d.f. = n - 1

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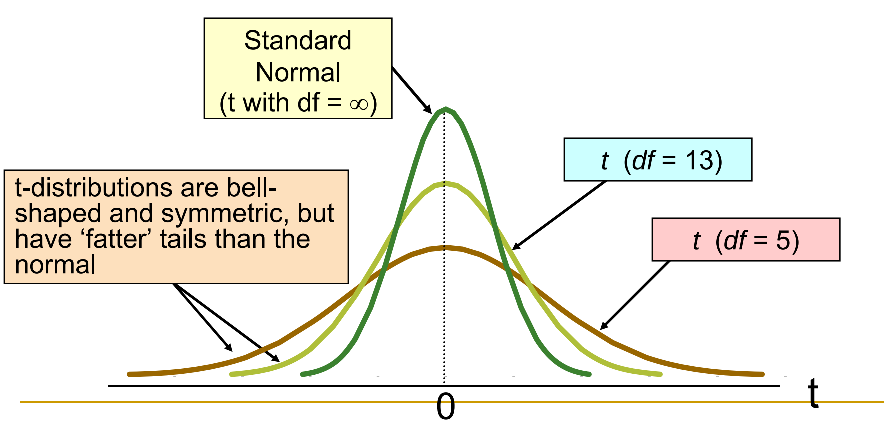
Student's t Distribution

Note: t ⟶ z as n increases



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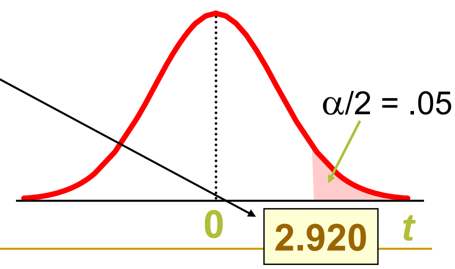
Student's t Table

	Upper Tail Area		
df	.25	.10	.05
1	1.000	3.078	6.314
2	0.817	1.886	2.920
3	0.765	1.638	2.353

The body of the table contains t values, not probabilities

Let: n = 3
df = n - 1 = 2
α = .10
$\alpha/2$ =.05



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t distribution values

With comparison to the z value

Confidence Level	t (10 d.f.)	t (20 d.f.)	t (30 d.f.)	z
.80	1.372	1.325	1.310	1.28
.90	1.812	1.725	1.697	1.64
.95	2.228	2.086	2.042	1.96
.99	3.169	2.845	2.750	2.58

Note: $t \rightarrow z$ as n increases

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Example

A random sample of n = 25 has $\bar{x}$ = 50 and s = 8. Form a 95% confidence interval for μ

Approximation for Large Samples

- Since t approaches z as the sample size increases, an approximation is sometimes used when n ≥ 30:

Correct formula

$$\bar{X} \pm t^{(n-1)}_{(1-\alpha/2)} \frac{s}{\sqrt{n}}$$

Approximation for large n

$$\bar{X} \pm z_{(0.5-\alpha/2)} \frac{s}{\sqrt{n}}$$

Determining Sample Size

- The required sample size can be found to reach a desired half width (H) and level of confidence (1 - α)
  - Required sample size, σ known:

$$n = \frac{z^2_{(0.5-\alpha/2)}\ \sigma^2}{H^2} = \left(\frac{z_{(0.5-\alpha/2)}\ \sigma}{H}\right)^2$$

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Determining Sample Size

- The required sample size can be found to reach a desired half width (H) and level of confidence (1 - α)
  - Required sample size, σ unknown:

$$n = \frac{z^2_{(0.5-\alpha/2)}\ s^2}{H^2} = \left(\frac{z_{(0.5-\alpha/2)}\ s}{H}\right)^2$$

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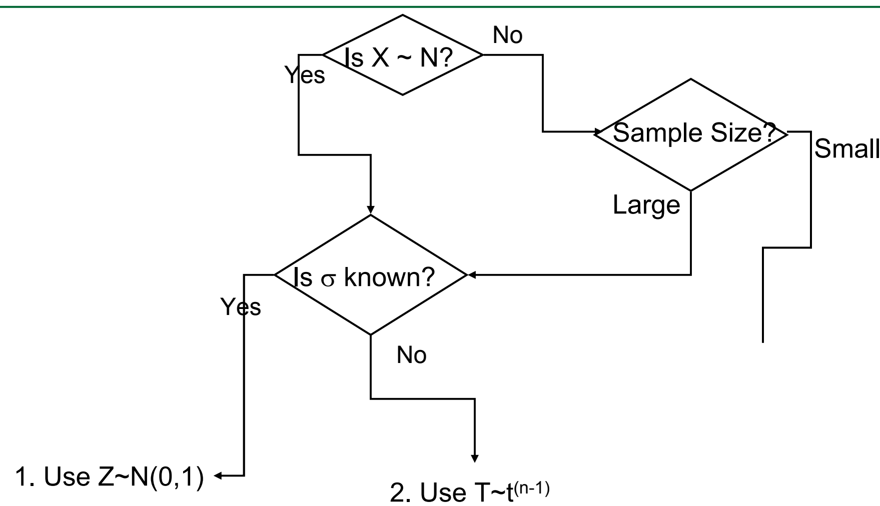
Required Sample Size Example

If σ = 45, what sample size is needed to be 90% confident of being correct within ± 5?

Confidence Interval Estimates



Confidence Intervals (1-α)%

1. Standard Normal

$$\text{Two-sided}: \overline{X} \pm Z_{(0.5-\alpha/2)} \frac{\sigma}{\sqrt{n}}$$

$$\text{One-sided Upper}: \mu \leq \overline{X} + Z_{(0.5-\alpha)} \frac{\sigma}{\sqrt{n}}$$

$$\text{One-sided Lower}: \mu \geq \overline{X} - Z_{(0.5-\alpha)} \frac{\sigma}{\sqrt{n}}$$

2. T distribution

$$\text{Two-sided}: \overline{X} \pm t^{(n-1)}_{(1-\alpha/2)} \frac{s}{\sqrt{n}}$$

$$\text{One-sided Upper}: \mu \leq \overline{X} + t^{(n-1)}_{(1-\alpha)} \frac{s}{\sqrt{n}}$$

$$\text{One-sided Lower}: \mu \geq \overline{X} - t^{(n-1)}_{(1-\alpha)} \frac{s}{\sqrt{n}}$$

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YDI 10.17

A beverage dispensing machine is calibrated so that the amount of beverage dispensed is approximately normally distributed with a population standard deviation of 0.15 deciliters (dL).

- Compute a 95% confidence interval for the mean amount of beverage dispensed by this machine based on a random sample of 36 drinks dispensing an average of 2.25 dL.
- Would a 90% confidence interval be wider or narrower than the interval above.
- How large of a sample would you need if you want the width of the 95% confidence interval to be 0.04?

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YDI 10.18

A restaurant owner believed that customer spending was below the usual spending level. The owner takes a simple random sample of 26 receipts from the previous weeks receipts. The amount spent per customer served (in dollars) was recorded and some summary measures are provided:

$n = 26$, $\bar{X} = 10.44$, $s^2 = 7.968$

- Assuming that customer spending is approximately normally distributed, compute a 90% confidence interval for the mean amount of money spent per customer served.
- Interpret what the 90% confidence interval means.