A GENERALIZED MODEL FOR EVALUATING SUPPLY CHAIN DELIVERY PERFORMANCE

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Abstract
Supply chain management has evolved as a core component in an organization's overall strategy for gaining and maintaining competitive advantage. The supply chain management philosophy serves as a foundation for integrating and effective managing value-adding activities such as raw material acquisition, production processing and physical distribution. In support of this philosophy, quantitative models for evaluating delivery performance to the end customer in the supply chain are used to assist managers in assuring that high levels of customer service and satisfaction are maintained.

In this paper we present a generalized delivery performance model that overcomes two existing limitations found in delivery performance models that have been reported in the literature. The model presented herein uses the gamma probability density function (pdf) in the construction of a cost based delivery performance model. The model is illustrated using a numerical example.

Key Words: *Delivery performance*, *Supply chain management*, *Cost based modeling*

1. Introduction

Supply chain management serves as the foundation of an organization's overall competitive strategy for attaining and maintaining competitive advantage. Under the supply chain management philosophy, value-adding activities such as raw materials acquisition, production processing and physical distribution are all coordinated to insure that customer demand is met with a correct order quantity that is delivered in a timely manner.

The need for performance measurement and evaluation in supply chain management is well recognized in the literature. Detailed discussions on the importance of performance measurement in supply chains may be found in Gunasekaran and Kobu (2007) and Lockamy and McCormack (2004). A taxonomy of useful supply chain performance metrics that spans the strategic, tactical and operational levels of supply chain operation has been effectively summarized in Bhagwat and Sharma (2007) and Gunasekaran et al

(2004). Within the hierarchy of supply chain performance metrics, delivery performance is acknowledged as a key metric for supporting operational excellence of supply chains (Gardner, 2010) and is classified as a strategic level performance measure by Gunasekaran et al (2004).

In today's competitive business environment customer dissatisfaction resulting from untimely delivery is of high concern to managers. Numerous empirical studies have documented the high level of performance that on time delivery plays in the operation of the supply chain (see for example, da Silveira and Arkader, 2007 and Iyer et al. 2004). In support of this concern, models for evaluating delivery performance to the final customer within multi-stage supply chains have been proposed by several researchers. These models and the delivery performance measures that they contribute serve as an integral precursor for managing improvements in delivery performance.

In this paper we develop a cost based delivery performance model in which delivery times are modeling using the gamma pdf. Our model contributes to the literature in that it: i) overcomes a common limitation that is inherent to delivery models found in the literature, and ii) represents a generalized modeling approach to the evaluation of supply chain delivery performance.

This paper is organized as follows. In Section 2 we provide a review of the literature on models of delivery performance and identify a common limitation inherent to these models. In Section 3 we introduce the mathematical form of a generalized model for evaluating supply chain delivery performance that overcomes the limitation of models currently found in the literature and demonstrate the model. In Section 4, we summarize our findings and discuss directions for future research.

2. Literature Review

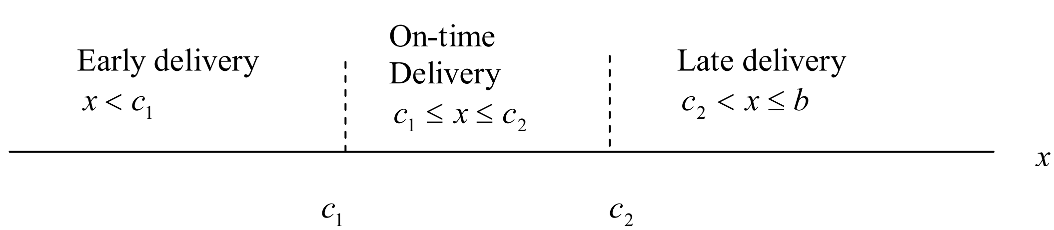
Models for evaluating delivery performance within supply chains can be categorized into two groups: i) index based models, and ii) cost based models. Table 1 provides an overview of this literature.

Table 1: Classification of Supply Chain Delivery Performance Models.

Index Based Models	Cost Based Models
Nabhani and Shokri (2009)	Shin et al. (2009)
Wang and Du (2007)	Guiffrida and Jaber (2008a)
Choudhary et al. (2006)	Guiffrida et al. (2008b)
Garg et al. (2006)	Guiffrida and Nagi (2006a)
	Guiffrida and Nagi (2006b)

Both categories of models are similar in that delivery timeliness to the final customer is analyzed with regard to the customer's specification of an on time delivery window. Under the concept of a delivery window, contractually agreed upon benchmarks in time are used to classify deliveries as being early, on time, and late (see Figure 1).

Figure 1. Illustration of Delivery Window.



Legend: x = random variable defining delivery time
c_1, c_2 = benchmark times defining early, on-time, and late delivery

All the models defined in Table 1 use the delivery window to assess the probabilities of early, on time and late delivery. The models differ in how they report delivery performance in terms of an overall metric. Index based models translate the probability of untimely (early and late delivery) into a “delivery capability index” measure which is similar to the family of process capability indexes that have been traditionally used in statistical process control activities in manufacturing. Cost based models translate the probability of untimely delivery into an expected cost measure.

The two classes of models are elegant in their application of statistical theory to the task of evaluating supply chain delivery performance; however, these models have two major limitations. First, the models define delivery time as a Gaussian random variable. By definition, a Gaussian random variable is defined over the range of negative infinity to positive infinity. In reality, a random variable for representing delivery time can only be defined from zero to positive infinity. As demonstrated in Guiffrida and Jaber (2006), using the Gaussian to model a non-negative random variable can lead to significant underestimation when the coefficient of variation exceeds 0.25. Second, by using the Gaussian pdf the models are restricting the delivery distribution to be symmetric. We argue that a modeling delivery time using the Gamma pdf offers a more exact and generalized representation for measuring supplies chain delivery performance. A gamma random variable can only take on positive values hence it is naturally suited for measuring delivery times which can not be negative. Since the gamma pdf is not constrained to be symmetric it supports a more generalized modeling approach for a delivery distribution.

3. Model Development

Consider an *n*-stage serial supply chain where an activity at each stage contributes to the overall delivery time to the final customer. Delivery time to the final customer is defined to be the elapsed time from the dispatch of an order by the originating supplier in the supply chain to the receipt of the product ordered by the final customer in the supply chain. Delivery lead time is composed of a series of activities (manufacturing, processing and transportation) at each stage of the supply chain. The activity duration of stage *i*, W_i, is defined by pdf $f_W(w,\theta)$ that is reproductive under addition with respect to parameter set θ. Delivery time to the final customer, $X = \sum_{i=1}^{n} W_i$, is defined by the resulting n-fold convolution of $f_W(w,\theta)$ which has pdf $f_X\left(x, \sum_{i=1}^{n}\theta_i\right)$. We assume that activity durations (manufacturing, processing and transportation time) at each stage of the supply chain independent.

For stage $i(i = 1,\ldots,n)$ let $W_i \sim Gamma(\alpha, k_i)$ with pdf

$$f_W(w) = \frac{\alpha^k e^{-\alpha w} w^{k-1}}{\Gamma(k)} \qquad (1)$$

for $w > 0$, scale parameter $\alpha > 0$, and shape parameter $k > 0$.

The mathematical form of the pdf governing delivery time $X = W_1 + W_2 + \ldots + W_n$ can be determined using moment generating functions.

For the gamma distribution the parameter set $\theta = \{\alpha, k\}$ and the corresponding moment generating function is

$$M_W(t) = \left(\frac{\alpha}{\alpha - t} \right)^k . \tag{2}$$

Hence, for $X = W_1 + W_2 + ... + W_n$,

$$M_X(t) = \left(\frac{\alpha}{\alpha - t} \right)^{k_1} \left(\frac{\alpha}{\alpha - t} \right)^{k_2} \lambda \left(\frac{\alpha}{\alpha - t} \right)^{k_n}$$

$$= \left(\frac{\alpha}{\alpha - t} \right)^{\sum_{i=1}^{n} k_i} . \tag{3}$$

Examining $M_X(t)$, we recognize, $X \sim Gamma\left(\alpha, \sum_{i=1}^{n} k_i \right)$.

The general structure of the cost based delivery model as defined by Guiffrida and Nagi (2006a) is

$$Y = QH \int_{-\infty}^{c_1} (c_1 - x) f_X(x) dx + K \int_{c_2}^{\infty} (x - c_2) f_X(x) dx \tag{4}$$

where Y = expected penalty cost per period for untimely (early and late) delivery
Q = constant delivery lot size
H = supplier's inventory holding cost per unit per unit time
K = penalty cost per time unit late (levied by the buyer)
c_1, c_2 = parameters defining the on time delivery window
$f_X(x)$ = the probability density function of delivery time.

We redefine (4) when $f_X(x)$ is gamma distributed thereby overcoming the two model limitations that were identified in the literature review.

Consider a two-stage supply chain in which each individual stage activity time is gamma distributed with common scale parameter α and shape parameters k_1 and k_2 respectively. Using (3), the pdf of delivery time $X = W_1 + W_2$ is gamma distributed and is defined by the random variable $X \sim Gamma(\alpha, k_1 + k_2)$. Restating (4) with $f_X(x)$ defined in terms of the gamma pdf leads to (see Appendix I for derivation)

$$Y = QH\left[\frac{c_1\gamma(k,\alpha c_1)}{\Gamma(k)} - \frac{\gamma(k+1,\alpha c_1)}{\alpha\Gamma(k)}\right] + K\left[\frac{\Gamma(k+1,\alpha c_2)}{\alpha\Gamma(k)} - \frac{c_2\Gamma(k,\alpha c_2)}{\Gamma(k)}\right]. \quad (5)$$

The advantage of using the gamma based delivery performance model defined by (5) over the Gaussian model is illustrated in the two figures below. First, as seen in Figure 2, the gamma based model is more general than the Gaussian model (G) in its application since it does not require that the delivery distribution exhibit symmetry. Second, as illustrated in Figure 3, the gamma model can be applied to evaluate the cost of untimely delivery for all possible values of the mean and variance of delivery time where the Gaussian model is not. For the range of values for the mean and variance displayed in Figure 3, a Gaussian based delivery model would underestimate the true total cost of untimely delivery as a result of the high probabilities of negative delivery times that would occur when $\mu < 4\sigma$.

Figure 2. Comparison of Gamma (G) and Gaussian Based Models in the Skewness (γ_1) and Excess Kurtosis Plane (γ_2).

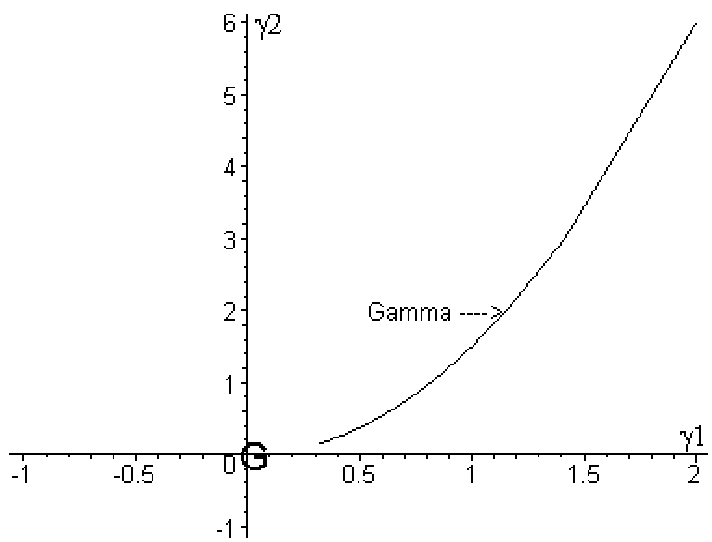
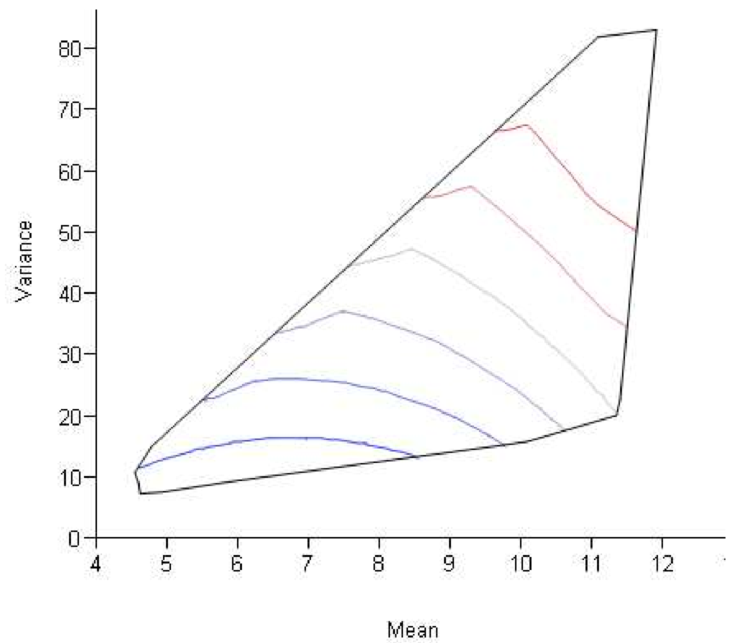


Figure 3. Cost Surface for Gamma Model for K/QH = 2, $C_1 = 5$ and $C_2 = 10$.



4. Summary and Directions for Future Research

In this paper we have presented a model for evaluating the expected cost of untimely delivery to the end customer in a supply chain when delivery times are gamma distributed. The model presented herein overcomes two limitations in the Gaussian based models found in the literature: i) the gamma model is applicable for a wide range of delivery distributions and is not constrained to be symmetric, and ii) the gamma model is defined for only positive delivery values and hence can be used to model delivery distributions in which a Gaussian model can not be applied.

There are several aspects of this research that can be extended. First, we can expand the scope of the model to include multiple products. Second, we could relax the assumption of independence among stage activity times. Lastly, we could determine the optimal width and placement of the delivery window within the delivery distribution.

References:

Bhagwat, R. and Sharma, M. K. (2007) "Performance measurement of supply chain management: A balanced scorecard approach", *Computers and Industrial Engineering*, 53(1), 43-62.

Choudhary, A. K., Singh, K. A. and Tiwari, M. K. (2006). "A statistical tolerancing approach for design of synchronized supply chains", *Robotics and Computer-Integrated Manufacturing*, 22, 315-321.

da Silveira, G. J. C. And Arkader, R. (2007). "The direct and mediated relationships between supply chain coordination investments and delivery performance", *International Journal of Operations & Production Management*, 27(2), 140-158.

Garg, D., Naraharai, Y. and Viswanadham, N. (2006). "Achieving sharp deliveries in supply chains through variance reduction", *European Journal of Operational Research*, 171(1), 227-254.

Gartner.com (2010) "The AMR Supply Chain Top 25 for 2010" http://www/gartner.com/resources/201200/201212/the_amr_supply_chain_top_25_20.pdf.

Guiffrida, A. L. and Jaber, M. Y. (2008a). "Managerial and economic impacts of reducing delivery variance in the supply chain", *Applied Mathematical Modeling*, 32(10), 2149-2161.

Guiffrida, A. L. and Jaber, M. Y. (2006). "The expected quantity short per replenishment cycle for truncated normal lead time demand", *Journal of Academy of Business and Economics*, 6(2), 88- 95.

Guiffrida, A. L., Jaber, M. Y. and Rzepka, R. A. (2008b). "An economic model for justifying the reduction of delivery variance in an integrated supply chain", *INFOR: Information Systems and Operational Research*, 46(2), 147-153.

Guiffrida, A. L. and Nagi, R. (2006a). "Cost characterizations of supply chain delivery performance", *International Journal of Production Economics*, 102(1), 23-26.

Guiffrida, A. L. and Nagi, R. (2006b). "Economics of managerial neglect in supply chain delivery performance", *The Engineering Economist*, 51(1), 1-17.

Gunasekaran, A. and Kobu, B. (2007) "Performance measures and metrics in logistics and supply chain management: a review of recent literature (1995-2004) for research and application", *International Journal of Production Research*, 45(12), 2819-2840.

Gunasekaran, A., Patel, C. and McGaughey (2004) "A framework for supply chain performance measurement", International *Journal of Production Economics*, 87(3), 333-347.

Iyer, K. N. S., Germain, R. and Frankwick, G. L. (2004). Supply chain B2B e-commerce and time-based delivery performance", *International Journal of Physical Distribution & Logistics Management*, 34(7/8), 645-661.

Lockamy, A. and McCormack, K. (2004) "Linking SCOR planning practices to supply chain performance, *International Journal of Production Research*, 47(9), 2449-2460.

Nabhani, F. and Shokri, A. (2008). "Reducing the delivery lead time in a food distribution SME through the implementation of six sigma methodology", *Journal of Manufacturing Technology*, 20(7), 957-974.

Shin, H., Benton, W. C. and Jun, M. (2009). "Quantifying suppliers' product quality and delivery performance: A sourcing policy decision model", *Computers and Operations Research*, 36, 2462-2471.

Wang, F. K. and Du, T. (2007). "Applying capability index to the supply chain network analysis", *Total Quality Management*, 18(4), 425-434.

Appendix I

When delivery time is gamma distributed the expected penalty cost for late delivery is

$$Y_{late} = K\int_{c_2}^{\infty}(x-c_2)f_X(x)dx \tag{A1}$$

$$= K\left[\int_{c_2}^{\infty}\frac{\alpha^k x^k e^{-\alpha x}}{\alpha\Gamma(k)}\alpha dx - c_2\int_{c_2}^{\infty}\frac{(\alpha x)^{k-1}e^{-\alpha x}}{\Gamma(k)}\alpha dx\right]. \tag{A2}$$

Substituting $u = \alpha x$, $du = \alpha dx$ and changing the lower limit of the integral from $x = c_2$ to $u = \alpha x = \alpha c_2$ yields

$$Y_{late} = K\left[\int_{\alpha c_2}^{\infty}\frac{u^k e^{-u}}{\alpha\Gamma(k)}du - c_2\int_{\alpha c_2}^{\infty}\frac{u^{k-1}e^{-u}}{\Gamma(k)}du\right]. \tag{A3}$$

Examining (A3), it is noted that the integrals are of the form of the incomplete gamma function

$$\Gamma(a,x) = \int_{x}^{\infty}e^{-t}t^{a-1}dt\ . \tag{A4}$$

For the first integral in (A3), the parameters of (A4) are defined as $a-1=k$ and $x=\alpha c_2$, which gives $\Gamma(k+1,\alpha c_2)$. For the second integral in (A3), $a-1=k-1$ and $x=\alpha c_2$, which gives $\Gamma(k,\alpha c_2)$.

Hence,

$$Y_{late} = K\left[\frac{\Gamma(k+1,\alpha c_2)}{\alpha\Gamma(k)} - \frac{c_2\Gamma(k,\alpha c_2)}{\Gamma(k)}\right]. \tag{A5}$$

The expected earliness cost is defined as

$$Y_{early} = QH\int_{-\infty}^{c_1}(c_1-x)f_X(x)dx \tag{A6}$$

$$= QH\left[c_1\int_0^{c_1}\frac{\alpha^k x^{k-1}e^{-\alpha x}}{\Gamma(k)}dx - \int_0^{c_1}\frac{x\alpha^k x^{k-1}e^{-\alpha x}}{\Gamma(k)}dx\right]. \tag{A7}$$

Substituting $u=\alpha x$, $du=\alpha dx$ and changing the upper limit of the integral from $x=c_1$ to $u=\alpha x=\alpha c_1$ yields

$$Y_{early} = QH\left[c_1\int_0^{\alpha c_1}\frac{u^{k-1}e^{-u}}{\Gamma(k)}du - \int_0^{\alpha c_1}\frac{u^k e^{-u}}{\alpha\Gamma(k)}du\right]. \tag{A8}$$

Examining (A8), the integrals are of the form of the incomplete gamma function

$$\gamma(a,x) = \int_0^x e^{-t}t^{a-1}dt\ . \tag{A9}$$

For the first integral in (A8), the parameters of (A9) are defined as $a-1=k-1$ and $x=\alpha c_1$, which gives $\gamma(k,\alpha c_1)$. For the second integral in (A8), $a-1=k$ and $x=\alpha c_1$, which gives $\gamma(k+1,\alpha c_1)$. Hence,

$$Y_{early} = QH\left[\frac{c_1\gamma(k,\alpha c_1)}{\Gamma(k)} - \frac{\gamma(k+1,\alpha c_1)}{\alpha\Gamma(k)}\right]. \tag{A10}$$

Combining (A5) and (A10) gives (5).