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Crossing of disclinations in nematic slabs

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Abstract. – It is shown experimentally that crossing and intercommutation of disclinations in a bounded nematic cell depend on surface orientation of the director and the relative strength of disclinations. Lines of opposite strength switch the pinned ends between the bounding plates and vanish independently of each other if the surface orientation is tangential. In contrast, tilted surface orientation preserves the stability of lines.

Introduction. – A common issue of current interest in cosmology [1]-[3] and condensed-matter physics [4]-[6] is the dynamics and interaction of topological defects. The problem is centered around the mechanisms of defect production during the transition into a broken-symmetry state and subsequent phase ordering. In condensed-matter systems, an additional point of interest is the defect-mediated mechanical properties of materials such as liquid crystalline polymers [7]-[11]. One of the key processes in the evolution of a network of linear defects is their crossing. Numerical simulation in both cosmology [12]-[14] and condensed-matter systems [15], [16] demonstrated the process of reconnection: two initial lines exchange parts as they cross so that each of the two ensuing lines has segments of the two original strings. Even in nematic liquid crystals, where any two singular lines are supposed to annihilate upon merger, one can observe this effect of reconnection. For example, as Chuang *et al.* [4] demonstrated, if two topologically stable lines disclinations cross in the nematic bulk, the result is a pair of two reconnected, but still topologically stable, lines.

So far both experimental [4], [17] ,[18] and theoretical [19], [20] studies of defect crossing have concentrated on the bulk feature. In this letter we explore how the boundedness of the system influences the result of crossing. The underlying idea is that the ensuing bulk lines can transform into the surface lines. If this is so then the final result of crossing should depend on the boundary conditions for the order parameter since the topological stability of defects in the bulk and at the boundary is generally different [21].

Experiment. – The nematic liquid crystal 5CB (EM Industries, Inc.) was used in all experiments at room temperature. Glass plates were spin-coated with dye-doped Polymethylmethacrylate (PMMA), supplied by IBM Almaden Research Center. Two coated plates of lateral size about 1 cm × 1 cm were assembled to form a flat cell with a gap fixed by thin Mylar©

(DuPont Co.) films. Although the cell thickness was not an essential factor in the experiment, it was optimized to 25 μm for the observational clarity. The PMMA material possesses two important features that make the experimental study of defect interaction possible.

First, the PMMA coating provides excellent *azimuthally degenerated tangential* orientation of the director **n** so that the topologically stable disclinations of strength $s = \pm 1/2$ are easy to create (for example, by cooling the system down from the isotropic melt). Two dark brushes emerge from the ends of disclinations terminating at the glass plates (so-called Schlieren texture, fig. 1 (a)). The brushes mark the regions where **n** is parallel either to the polarizer or the analyzer. The two complementary mutually perpendicular states of **n** can be discriminated by the use of a quartz edge. Rotation of the sample between the crossed polarizer allows us to reconstruct the whole director field around the lines and thus to establish their strength.

The second important feature is good adhesion of the ends of the disclinations to the PMMA substrates. The disclinations that connect the opposite plates remain stable against approaching each other and in-plane annihilation. Under a gentle shift of the top plate the corresponding ends of the lines follow the movement of the plate. This feature allowed us to study the line crossing by rotating the upper plate of the cell. The plate was rotated in such a manner that two initially vertical and parallel disclinations found themselves twisted in a crossed position. The ends of the lines remained pinned at the substrates, the crossing thus took place in the bulk of the cell.

We also prepared samples with spin-coated polyimide layers (Nissan Polyimide 610) that provide an alternative surface alignment with titled **n**. The polyimide layers were left nonrubbed to preserve azimuthal degeneracy. However, even nonrubbed polyimide has some local anisotropy and the azimuthal degeneracy is preserved only when averaged over areas of supramolecular size.

To check surface alignment, we measured the capacitance of the cells as the function of the magnetic field applied parallel to the bounding plates. For PMMA cells, there was no change in the capacitance as the field increased from 0 to 1 Tesla, indicating that PMMA does indeed impose tangential anchoring. For the polyimide cell, there is a drop in the capacitance value (5%). Since the dielectric anisotropy of 5CB is positive, this shows that the nonrubbed polyimide provides tilted alignment with the angle between **n** and the substrate about 5°. Strictly speaking, isolated disclinations that connect the opposite plates are impossible for tilted **n**; the lines should be accompanied by surface walls. Across these walls, the director tilt changes sign. Experiments reveal that such walls of weak optical contrast exist. However, for small tilt, the energy cost of the walls is small and one can still identify $|s| = \frac{1}{2}$ disclinations and perform cross-linking.

The scenarios of interactions were recorded through a polarizing microscope with a CCD camera.

Results. – Defect lines in nematic liquid crystals are characterized by a director rotation angle $2s\pi$ measured as one goes counter-clockwise around the defect core [22], where the strength s can be either integer or half-integer. Lines with an integer s are unstable: they can be smoothly transformed into a uniform state $s = 0$. All lines with half-integer s are stable and topologically equivalent to each other so that, for example, a line $s = 1/2$ can be smoothly transformed into a line $s = -1/2$. Thus, there is only one type of stable nematic disclinations. Despite the fact that the sign of s is topologically meaningless, it controls defect interactions (repulsion of alike signs *vs.* attraction of opposite signs [22]). The results below demonstrate that the sign of s also controls the crossing of disclinations. The scenario for the pair $(\frac{1}{2}\ ,\ -\frac{1}{2})$ is different from the scenario for the pair $(-\frac{1}{2}\ ,\ -\frac{1}{2})$.

Figure 1 shows the crossing scenario for the pair $(\frac{1}{2}\ ,\ -\frac{1}{2})$ in a tangentially anchored cell. When the initial lines that connect the opposite plates (fig. 1 (a)) are brought close in the

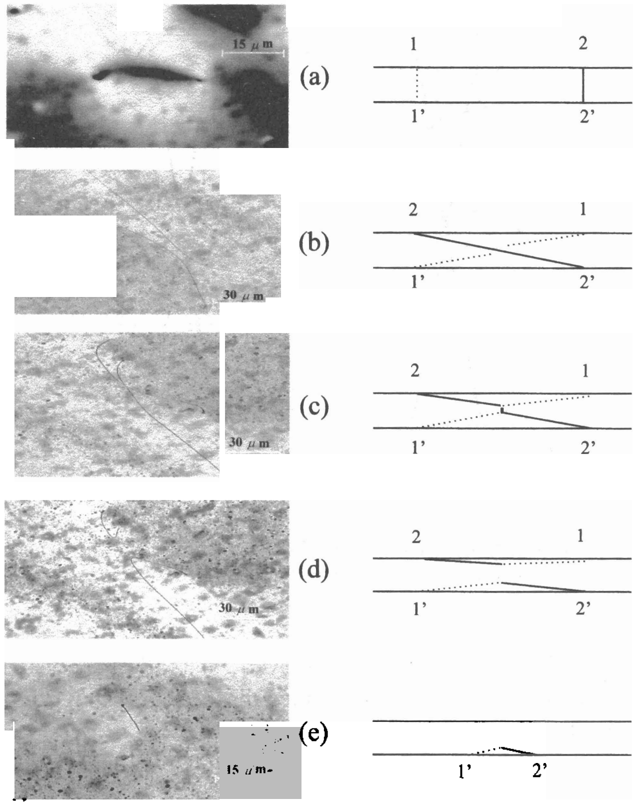

Fig. 1. – The actual images (left column) and schematic pictures (right column) of the crossing and reconnection in the case of $\frac{1}{2}$ and $-\frac{1}{2}$ lines. Picture (a) was taken with crossed polarizers while (b)-(e) were taken without them for clarity. Elapsed time measurement was started at (b). (a) Initial state of the pair. One can identify the charges $(+\frac{1}{2}\ ,\ -\frac{1}{2})$ by rotating the stage of the polarizing microscope. (b) The top plate is shifted. The disclination lines are elongated but are not yet crossed (0 s). (c) The two lines are crossed and form a junction (1.0 s). (d) After the reconnection, the two ensuing lines have their ends on the same plates (1.5 s). (e) One of the ensuing lines (located on the top plate) has disappeared due to the shortening, another shrinks and eventually disappears (3.0 s).

middle of the cell (fig. 1 (b)), their central parts attract each other to form a joint segment (fig. 1 (c)). The joint segment disappears, leaving two reconnection cusps. Each resulting line has both ends at the very same surface (fig. 1 (d)), in contrast to the original geometry. As the time elapses, the lines disappear independently of each other (fig. 1 (e)). The end portions of the line vanish because **n** reorients into a uniform state and the bulk portion shrinks. Hence the crossing of lines in a tangentially anchored slab leads to their elimination at the bounding surfaces.

The case of crossing of lines with the *same signs* $(-\frac{1}{2}\ ,\ -\frac{1}{2})$ for the same tangentially anchored cell is different. Although the attraction between the lines is not apparent, they also form a point of merger. The disclinations exchange ends, but this time each ensuing line still has two ends located at the opposite plates. Thus the final structure is identical to the original one: two stable lines that connect the opposite plates.

Tilted boundary conditions change the result of intercommutation of the pairs $(\frac{1}{2}\ ,\ -\frac{1}{2})$. Until the reconnection, the process is the same as in the PMMA cell with tangential orientation.

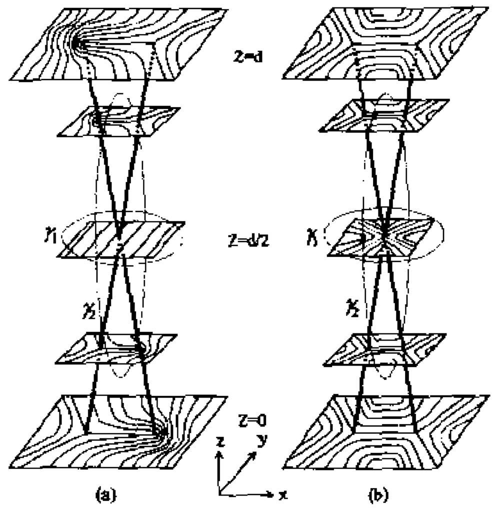

Fig. 2. - Schematic director configurations for crossing of pairs $(+\frac{1}{2}, -\frac{1}{2})$ (a) and $(-\frac{1}{2}, -\frac{1}{2})$ (b). Note the difference in the winding numbers in the mutually perpendicular paths γ_1 and γ_2.

However, no shortening of the final lines is observed, instead the lines transform into ***stable surface disclinations***. The core width of the surface disclination (few μms) is much wider than the core of the bulk disclination.

Discussion

a) Direction of reconnection. - As stated above, the direction of reconnection depends on the relative signs of s. The standard topological approach [3], [19], [20] is to consider two mutually orthogonal planes, in our case, parallel to the bounding plates and perpendicular to them, fig. 2. The closed loops γ_1 and γ_2 drawn in these two planes correspond to different total winding numbers $|s_t|$. For the pair $(\frac{1}{2}, -\frac{1}{2})$ the contour γ_1 encloses a configuration with $|s_t| = 0$, while γ_2 encloses a configuration with $|s_t| = 1$ (fig. 2(a)). For the pair $(-\frac{1}{2}, -\frac{1}{2})$, the situation is opposite: "horizontal" γ_1 corresponds to $|s_t| = 1$, while "vertical" γ_2 corresponds to $|s_t| = 0$ (fig. 2(b)). Comparing this with experiment, one concludes that the direction of reconnection is defined by $|s_t|$. The plane $|s_t| = 0$ crosses no disclinations in the final state, while the plane $|s_t| = 1$ crosses the ensuing lines. In other words, the dissection occurs in the plane $|s_t| = 0$.

The difference between $|s_t| = 1$ and $|s_t| = 0$ is not topological though: these states correspond to the same trivial element of the first homotopy group $\pi_1(RP^2) = Z_2$ of the nematic order parameter space which is the projective plane RP^2 (Z_2 is the group of two integers). The fact that the dissection never occurs in the plane $|s_t| = 1$ means that the transformation of $|s_t| = 1$ into $|s_t| = 0$ configuration is prevented by the energetics of the director field. Such a transformation reorients $\mathbf{n}$ along the vertical z-axis (fig. 2(b)); this process is hindered by tangential alignment ($\mathbf{n}$ is in the x, y plane) and residual in-plane flow.

b) Switching ends and elimination of defects. - The elimination of disclinations $(\frac{1}{2}, -\frac{1}{2})$ after crossing in the cell with tangential alignment is an interesting consequence of different topological stability of defects in the bulk and at the surface [21]. Two initial lines connect the opposite plates and can annihilate only by approaching each other; pinning of the ends at the bounding surfaces prevents this process if the lines are sufficiently separated. In contrast, the lines that emerge after crossing have both their ends at the *very same surface*. Depending on the director alignment at the surface, the surface parts of these lines can become topologically unstable.

The stability of the surface disclinations is given by the relative homotopy group $\pi_1(R, \overline{R})$ [21] where $R = RP^2$ and $\overline{R}$ is the order parameter space at the surface. For in-plane degenerate alignment, $\overline{R}$ depends on the angle α between $\mathbf{n}$ and the normal to the surface [23]:

$$\overline{R} = \begin{cases} 0, & \alpha = 0\,, \\ S^1, & 0 < \alpha < \dfrac{\pi}{2}\,, \\ S^1/Z_2, & \alpha = \dfrac{\pi}{2}\,; \end{cases}$$

here S^1 is the circle and S^1/Z_2 is the circle with antipodal points identified. The group $\pi_1(RP^2, S^1/Z_2) = 0$ is trivial: *no stable disclinations exist on the surface with tangentially degenerate* $\mathbf{n}$. When a stable line $|s| = \frac{1}{2}$ comes from the bulk, it disappears at such a surface through director reorientation along the disclination core (*i.e.* "escapes in the third dimension" [22]).

In contrast, tilted boundary conditions preserve the stability of disclinations: with $\overline{R} = S^1$, one has $\pi_1(RP^2, S^1) = \pi_1(RP^2) = Z_2$ [23] and the surface disclinations are described by the elements of the same group as the bulk disclinations. The only difference would be in the structure and the energy of the core. The core radius l_w of the surface disclinations is defined by the surface anchoring coefficient W and the elastic constant K [24]: $l_w \sim \frac{K}{W}$. As a result, l_w is macroscopic, $l_w \sim 1$–$10\,\mu$m and thus significantly larger than the core radius of the bulk line, $r_c \sim 10$ nm [25]. Consequently, the energy of the surface line is smaller than the energy of the bulk line by a factor $\sim \ln(\frac{l_w}{r_c})$ [24].

Finally note another peculiarity of crossing related to the elastic energy. For tangential alignment, the elastic energy before the crossing is due to the interaction between two disclinations s_1 and s_2: $U_{in} = \pi s_1 s_2 K d \ln(\frac{x}{r_c})$, where x is the separation along the bounding plates and d is the thickness of the cell and simultaneously the length of each line. The crossing intercommutes not only the ends of the lines but also the characteristic lengths x and d and the energy of each ensuing line, while in the bulk is $U_f \simeq \pi s^2 K x \ln(\frac{d}{r_c})$. Thus $U_f \gg U_{in}$, since $x \gg d$ in the experiment. Although the pinning force can prevent the movement and annihilation of the initial lines, it might be too weak to prevent the shortening of the bulk segments of the lines after the crossing.

To conclude, we observed crossing and reconnection of disclinations in a bounded nematic cell. The result of crossing depends on the disclination strength and boundary conditions. When two lines of opposite sign cross in a cell with tangential degenerate alignment, they intercommute the ends and eventually both disappear at the bounding surface. This defect elimination through cross-linking can find practical applications as a tool to control the defect densities in liquid crystal materials. Tilted boundary conditions preserve the topological stability of the ensuing lines.

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