Chapter 6 Modeling Random Events: The Normal and Binomial Mode

Content:

6.1 Probability Distributions are Models of Random Experiments

A **probability model (simulation)** describes how we think that data are produced**.**

A **probability distribution** also called **probability distribution function** (pdf) must meet these conditions:

1) It describes all possible outcomes that are **mutually exclusive** (the only one will happen);
   We can denote random outcome as X   Outcomes can be events in some other sample space.
2) Each outcome has a probability from the interval [0, 1]. Shorthand: $\mathbf{0 \le P(X) \le 1}$
3) **The sum of probabilities for all outcomes must be 1.**

Note: In statistical literature **pdf** often stands for **probability density function** which is probability distribution function for **continuous variables**. In the "Essential Statistics" by Gould and Ryan **pdf** stands for **probability distribution function** (probability distribution for any type of variable).

Example (distribution of the categorical data):
Suppose your iPod playlist has 10 songs on it: 6 are rock, 2 are country, 1 is pop, and 1 is hip-hop.
If you put your player on shuffle, what is the probability that the first song to play is a rock?

Create the probability distribution function (pdf) for this scenario:

1. determine all outcomes
2. determine the probability for each outcome
3. verify that probabilities add up to 1

Probability distribution function table:

Outcome: X	Probability: P(X)
Rock	$\frac{6}{10}$
Not Rock	$\frac{4}{10}$

$$\frac{6}{10}+\frac{4}{10}=1$$

Determine the probability for each outcome:

Use equiprobable space in which outcomes "Rock" and "Not Rock" are events:

S={R1, R2, R3, R4, R5, R6, C1, C2, P1, H1}   N=10

Probability of each simple (elementary) event is $\frac{1}{10}$

Event to select a rock is: Rock = {R1, R2, R3, R4, R5, R6} $\mathrm{P(Rock)} = \frac{6}{10}$

Event to select a non-rock is: "Not Rock" = { C1, C2, P1, H1 } $\mathrm{P(Not\ Rock)} = \frac{4}{10}$

Discrete vs. Continuous

Discrete outcomes are numerical values that can be listed or counted.
Continuous outcomes are numerical values that cannot be listed or counted because they **occur over a range**.

Identify the following as discrete or continuous
a. The amount of liquid consumed by the average American each day
b. The weight of newborns at a local hospital
c. How many pairs of shoes each person in the class owns
d. The number of pages in a standard math text book
e. The amount of electricity used daily in a home
f. The number of customers entering a restaurant in one-day
g. The speed of a train

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*18. Determine whether the random variable is discrete or continuous. In each case, state the possible values of the random variable.

(a) Is the number of people in a restuarant that has a capacity of 250 discrete or continuous?

- **A.** The random variable is continuous. The possible values are $0 \le x \le 250$.
- **B.** The random variable is discrete. The possible values are $0 \le x \le 250$.
- **C.** The random variable is continuous. The possible values are x = 0, 1, 2,..., 250
- **D.** The random variable is discrete. The possible values are x = 0, 1, 2,..., 250

(b) Is the time required to download a file from the Internet discrete or continuous?

- **A.** The random variable is continuous. The possible values are $t > 0$.
- **B.** The random variable is continuous. The possible values are t = 1, 2, 3, ...
- **C.** The random variable is discrete. The possible values are $t > 0$.
- **D.** The random variable is discrete. The possible values are t = 1, 2, 3, ...

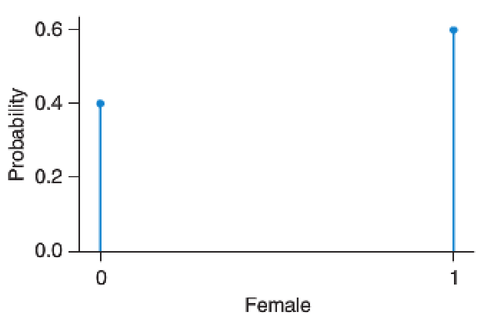
Discrete Probability Distributions can be represented as **Tables, Graphs or Equations**

Probability in discrete distribution is **relative frequency.**

An example of the graph and table:

A statistics class at UCLA was approximately 40% male and 60% female. Let's arbitrarily code the males as 0 and the females as 1. If we select a person at random, what is the probability that the person is female?

Creating a probability distribution for this situation is as easy as listing both outcomes (0 and 1) and their probabilities (0.40 and 0.60).



Female	Probability
0	0.40
1	0.60

When there are too many outcomes to show in the table we may use the formula.

Example:
The probability of having x children before the first girl is: $\mathrm{P(X)} = \left(\frac{1}{2}\right)^{x}$

1st is a girl probability: $\frac{1}{2}$

2nd is a girl probability: $\frac{1}{2} * \frac{1}{2} = \left(\frac{1}{2}\right)^2$

3rd is a girl probab.: $\frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \left(\frac{1}{2}\right)^3$

...

In discrete probabilities, the probability of each outcome X is in the interval [0,1]: $\mathbf{0 \leq P(X) \leq 1}$

Also, the sum of all probabilities is equal to 1. In short: $\sum \mathbf{P(X) = 1}$

The sum of 1 means that at least one of the outcomes must occur (we are 100% sure that something will happen).

*21. Determine whether the distribution is a discrete probability distribution.

x	10	20	30	40	50
P(x)	0.28	0.4	0.15	0.11	0.06

- **A.** No, because the sum of the probabilties is not 1.
- **B.** No, because one probability is greater than 1.
- **C.** Yes, because the probabilities meet all the conditions.
- **D.** No, because one probability is less than 0.

Work:

a) Are all events mutually exclusive?

b) Are all probabilities in [0,1]?

c) Do they add up to 1?

*22. Determine the required value of the missing probability to make the distribution a discrete probability distribution.

x	P(x)
3	0.27
4	?
5	0.30
6	0.26

(Type a P(4) = ______

Work:

Example: Determine probabilities, make a graph, and find the expected value.

Roll a fair six-sided die. A fair die is one in which each side is equally likely to end up on top. You will win \$4 if you roll a 5 or a 6. You will lose \$5 if you roll a 1. For any other outcome, you will win or lose nothing.

Outcome: X	**Probability: P(X)**
A: 4 dollars for 5 or 6	P(A) = $\frac{1}{3}$
B: -5 dollars for 1	P(B) = $\frac{1}{6}$
C: 0 dollars for 2, 3, or 4	P(C) = $\frac{1}{2}$

Make a graph

Determine probabilities based on equiprobable sample space:

S={1, 2, 3, 4, 5, 6 } N = 6

$A = \{5,6\} \quad P(A) = \frac{2}{6} = \frac{1}{3} \qquad B = \{1\} \quad P(B) = \frac{1}{6}$

$C = \{2,3,4\} \quad P(C) = \frac{3}{6} = \frac{1}{2}$

How much we can expect to win or lose?

$$\mathbf{E(X)} = \mathbf{4} * \frac{1}{3} - \mathbf{5} * \frac{1}{6} + 0 * \frac{1}{2} = \frac{8}{6} - \frac{5}{6} = \frac{3}{6} = \frac{1}{2}$$

We can expect to win 0.5 dollars (50 cents).

NEW CONCEPT: Expected value (mean of a discrete probability distribution) $E(X)$

is the weighted average of discrete values multiplied by their probabilities.

$$E(X) = \sum [X * P(X)]$$

E(X) represents the **"average" expected outcome** (expected winning or loss).

Hint: Create a table that shows the discrete probability distribution.

*25. In the game of roulette, a player can place a $6 bet on the number 32 and have a $\frac{1}{38}$ probability of winning. If the metal ball lands on 32, the player gets to keep the $6 paid to play the game and the player is awarded an additional $210. Otherwise, the player is awarded nothing and the casino takes the player's $6. What is the expected value of the game to the player? If you played the game 1000 times, how much would you expect to lose?

Outcome: X	**Probability: P(X)**
A: get $210 for 32	P(A) = $\frac{1}{38}$
B: lose $6 get other number	P(B) = $\frac{37}{38}$

B is complementary to A so: $P(B) = 1 - \frac{1}{38} = \frac{37}{38}$

$$E(X) = \sum [X * P(X)] = 210 * P(A) - 6 * P(B)$$

$$E(X) = 210 * \frac{1}{38} - 6 * \frac{37}{38} = \frac{210}{38} - \frac{222}{38} = \frac{-12}{38} \approx -0.31579$$

Interpret: It is expected that the player will lose $0.32 in each game

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If someone played 1000 games **the expected loss is**: 1000*(0.31579) = 315.79 dollars

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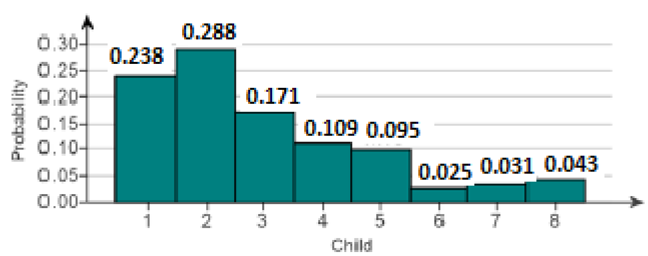
*23. The probability histogram to the right represents the number of live births by a mother 44 to 48 years old who had a live birth in 2012.



(a) What is the probability that a randomly selected 44- to 48-year-old mother who had a live birth in 2012 has had her fourth live birth?

(b) What is the probability that a randomly selected 44- to 48-year-old mother who had a live birth in 2012 has had her fourth or fifth live birth?

(c) What is the probability that a randomly selected 44- to 48-year-old mother who had a live birth in 2012 has had her sixth or more live birth?

(d) If a 44- to 48-year-old mother who had a live birth in 2012 is randomly selected, how many live births would you expect the mother to have had?

Continuous outcomes are numerical values that cannot be listed or counted because they **occur over a range** so their probabilities are defined for the range of outcomes as well.

Continuous probabilities are represented as areas under curves.

The curve is called **"probability density curve".** **The total area under the curve is 1.**

Example: Suppose you want to know the probability that you will wait in line for less than 2 minutes when you go to the coffee shop. You can't list all possible outcomes that could result from your visit: 1.0 minute, 1.00032 minutes, 2.00000321 minutes.

To find the probability of waiting between 0 and 2 minutes, we find the area under the density curve and between 0 and 2 (Figure 6.6).

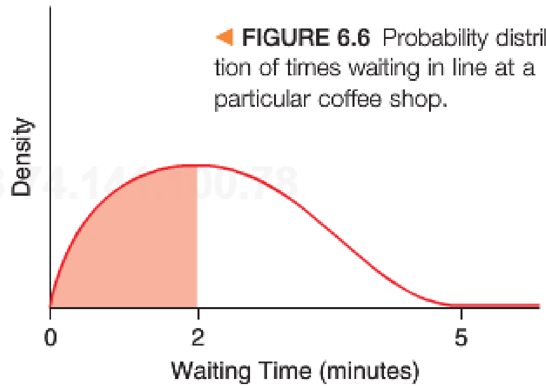
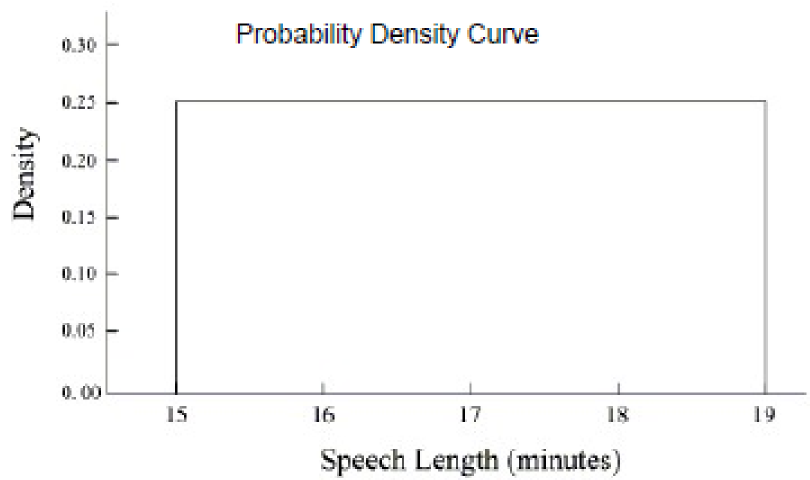


FIGURE 6.6 Probability distribution of times waiting in line at a particular coffee shop.

The curve may have a different shape. Even become a rectangle as in the example below.

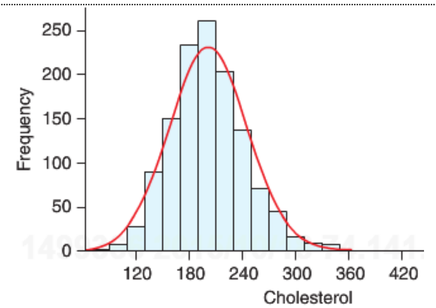
15. At a course in public speaking, the instructor always gives an opening speech that lasts between fifteen and eighteen minutes. The length of the speech can be modeled by a uniform distribution, that is, the speech is just as likely to last fifteen minutes as it is to last eighteen minutes. The probability density curve is shown below. What is the probability that the speech will last at least seventeen minutes? What is the probability that the speech will last between fifteen and sixteen minutes?

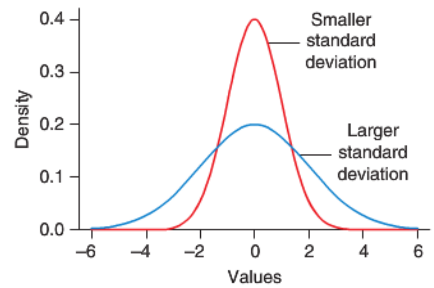
- A. 0.25; 0.50
- B. 0.50; 0.75
- C. 0.50; 0.25
- D. This cannot be determined

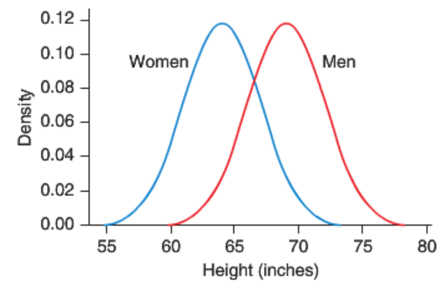


6.2 The Normal Model (for bell-shaped, symmetrical, and unimodal distributions)

Two-thirds of distributions are symmetrical and unimodal.
We use the Normal model to model these distributions and in most cases variables are continuous.







Properties of a normal distribution:
1) A normal curve is ______________,

__________________ and bell-shaped.

2) A normal curve is completely defined by its mean ____ and standard deviation_____.

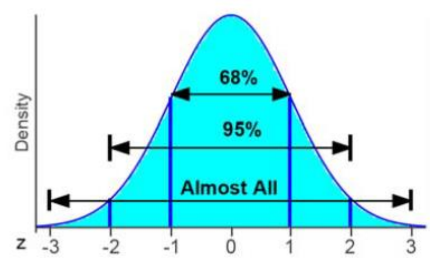
3) The total area under the normal curve equals ____.

The notation: **N(μ, σ)**

The normal curve N(64, 3) has a mean ____ and standard deviation____________

Each normal distribution can be turned into

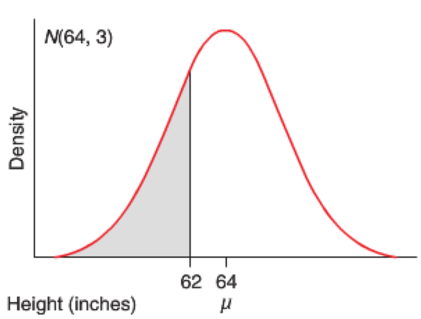
Standard Normal Distribution N(0, 1) by calculating z-scores

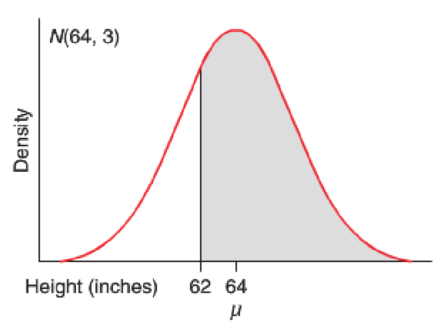


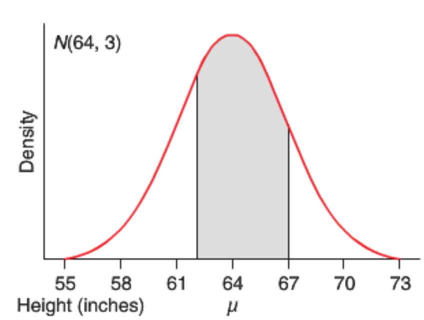
a) Roughly what percentage of z-scores is between -2 and 2?
b) Roughly what percentage of z-scores is between -3 and 3?
c) Roughly what percentage of z-scores is between -1 and 1?
d) Roughly what percentage of z-scores are more than 0?

Continuous probabilities are represented as areas under curves. In other words: area = probability

The curve is called **"probability density curve". The total area under the curve is 1.**
Example:
For the N(64, 3) of women heights in the US, find the probability of selecting a woman:
a) less than 62 inches tall (same as less than or equal 62in; No difference between > and ≥)
b) taller than 62 inches (same as greater than or equal 62in)
c) tall between 62 and 67 inches

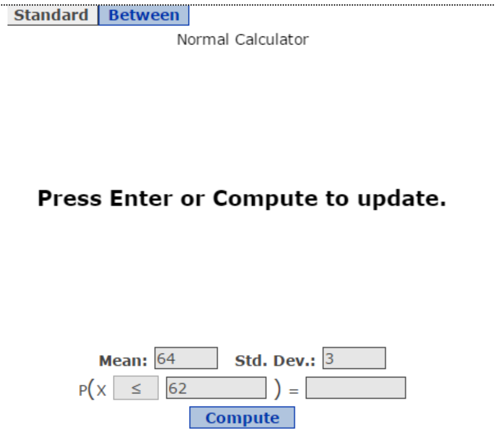


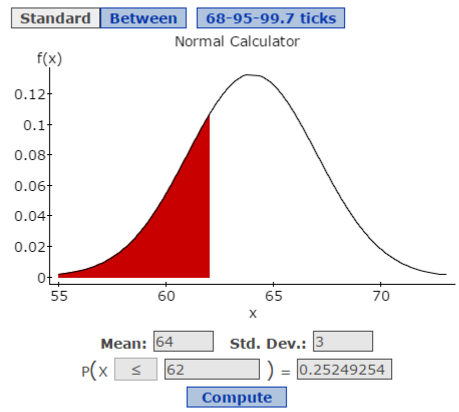




Find probabilities using StatCrunch for N(64, 3):

Stat -> Calculators -> Normal;   Enter the values that we know and press Compute





a) less than 62 inches tall

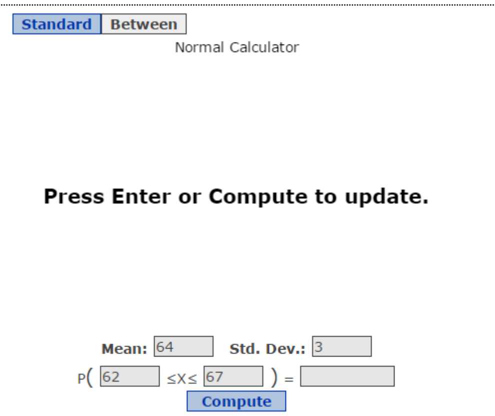
$$P(X < 62) = 0.2525 = 25.25\%$$

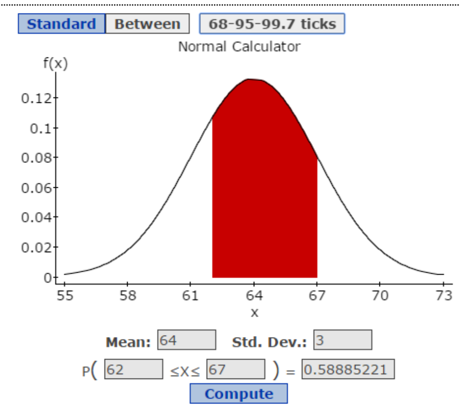
b) taller than 62 inches

This is a complement of $P(X > 62)$:

$$\mathbf{P(X > 62) = 1 - P(X < 62)}$$

$$\mathbf{P(X > 62) = 1 - 0.2525 = 0.7475 = 74.75\%}$$





c) tall between 62 and 67 inches

$$P(62 < X < 67) = 0.5889 = 58.89\%$$

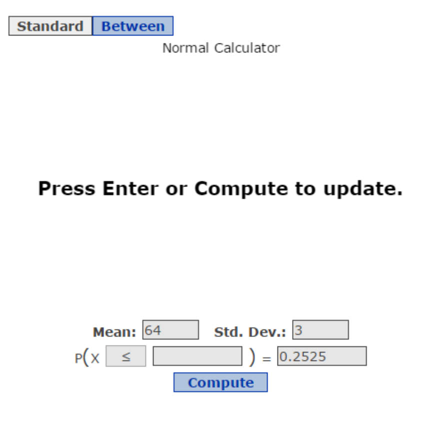
In this case “68-95-99.7 ticks “ were selected so they show in the graph.

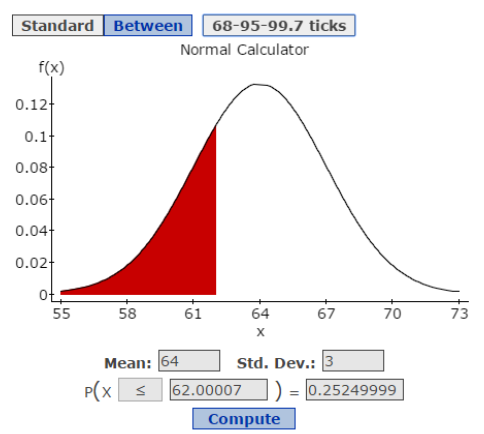
Interpretation:

a) Roughly 25.25% of women in the US are less than 62 inches tall.
b) Roughly 74.75% of women in the US are more than 62 inches tall.
c) Roughly 58.89% of women in the US are between 62 and 67 inches tall.

Find the measurements from percentiles using StatCrunch (inverse normal)

Example: For the N(64, 3) of women heights in the US, find the height of women who are in 25.25% (0.2525).

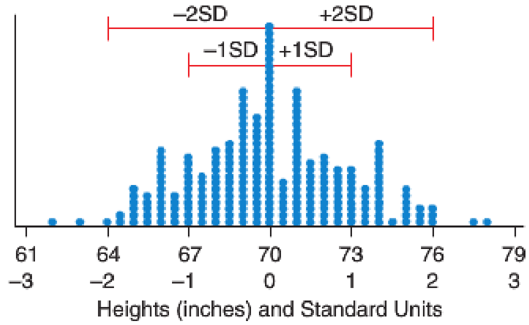




The result is 62 inches as expected.

Standard Normal Distribution and Probabilities in the table of Standard Normal Cumulative Probabilities

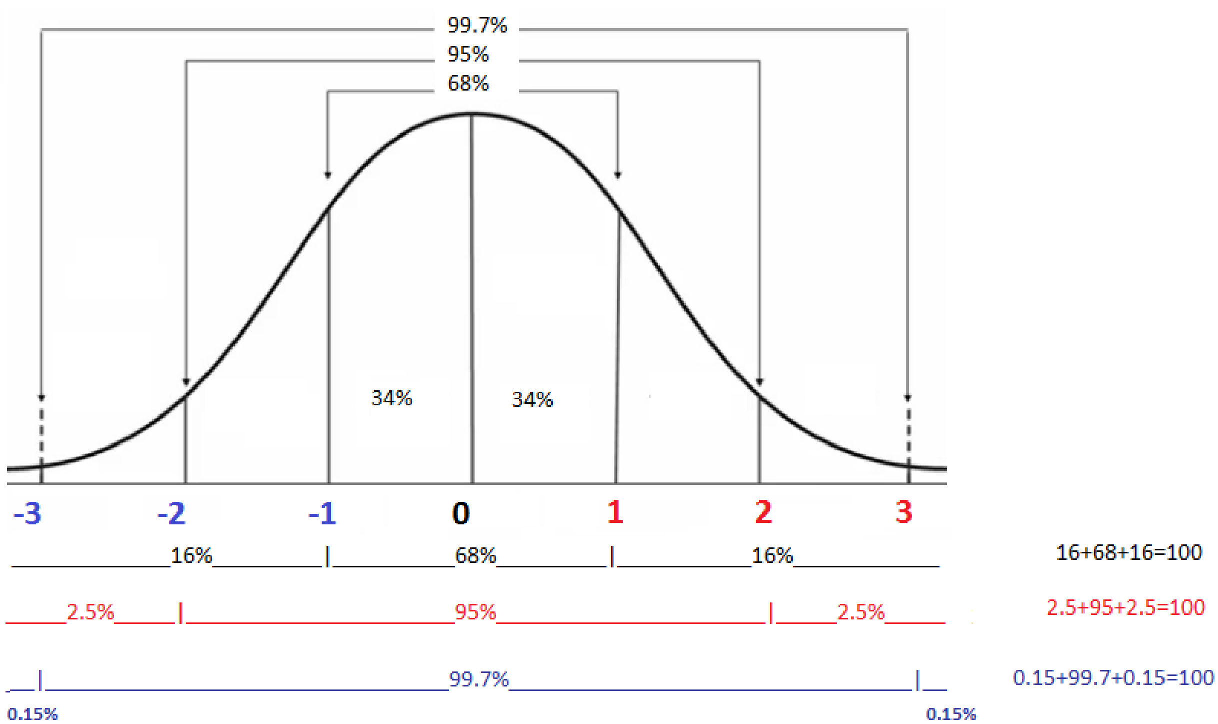
We saw earlier that by calculating z-scores any normal distribution is transferred into the ***standard*** *normal distribution.*



Standard Normal Distribution $\mathbf{N(0,1)}$. **The standard normal distribution has a** $\mu = 0$ **and** $\sigma = 1$

Scores on the x-axis are z-scores and they have values $(-\infty, \ \infty)$.

Z-score indicates how far is the score is from the mean in terms of standard deviation. For each z-score, we can compute probability using Normal calculator and entering Mean = 0 and Std. Dev = 1.



The process of transformation of normal distribution into **standard** normal distribution allows us to:

1. compare different normal distributions by turning scores into z-scores and comparing z-scores

2. in StatCrunch -> Calculators -> Normal, we can calculate the probability for each z-score (or we can use a table instead of StatCrunch)

3. in StatCrunch -> Calculators -> Normal, we can find the z-score for given probability (or we can use a table instead of StatCrunch)

Use StatCrunch to determine the area under the **standard normal curve** that lies:

a) To **the left** of $z = 0.64$ P(z < 0.64) = ___________

b) To **the left** of $z = -0.82$ P(z < -0.82) = ___________

c) To **the right** of $z = 1.49$ _________________

d) To **the right** of $z = -0.39$ __________________

Find probabilities using Standard Normal table

The standard normal table contains probabilities in terms of z-scores. Because of that, we must:

1. compute z-score first
2. read the probability from the table

Example:

For the N(64, 3) of women heights in the US, find the probability of selecting a woman less than 62 inches tall.

a) Compute z-score: $z(x) = \frac{x-\mu}{\sigma}$ so $z(62) = \frac{62-64}{3} = -\frac{2}{3} \approx -0.6667$

b) In the Standard Normal table, each z-score has only 2 decimals so we will round to : $z(62) \approx -0.67$

Table 2: Standard Normal Cumulative Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451

Due to rounding, the computed probability is: $P(62) = 0.2514 = 25.14\%$

Interpretation: About 25.14% of women in the US are less than 62 in tall.

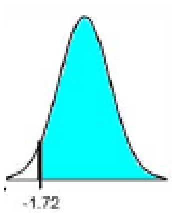
Use a table to find the indicated area under the standard Normal curve. Include an appropriately labeled sketch of the Normal curve and shade the appropriate region.

a. Find the area to the left of a z-score of − 1.72.

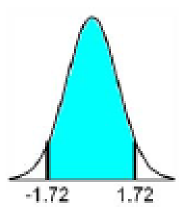
b. Find the area to the right of a z-score of − 1.72.

a. Which graph below shows the area in a standard Normal curve to the left of − 1.72?

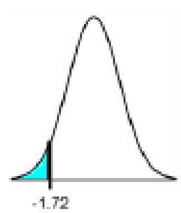
A.



B.



C.



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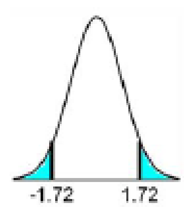
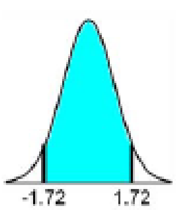


The total area to the left of z = − 1.72 is [ ].

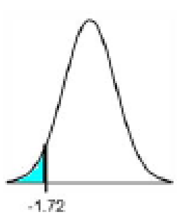
(Round to four decimal places as needed.)

b. Which graph below shows the area in a standard Normal curve to the right of − 1.72?

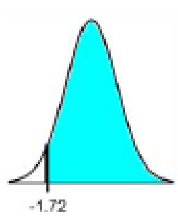
A.



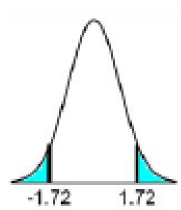
B.



C.



D.



The total area to the right of z = − 1.72 is [ ].

(Round to four decimal places as needed.)

Discrete variable but normally distributed.

A test has scores with a mean of 150 and a standard deviation of 25 and are approximately Normally distributed.

a. The test score at the 25th percentile is 133. What is the test score at the 75th percentile?

The 75th percentile is ______.
(Round to the nearest integer as needed.)

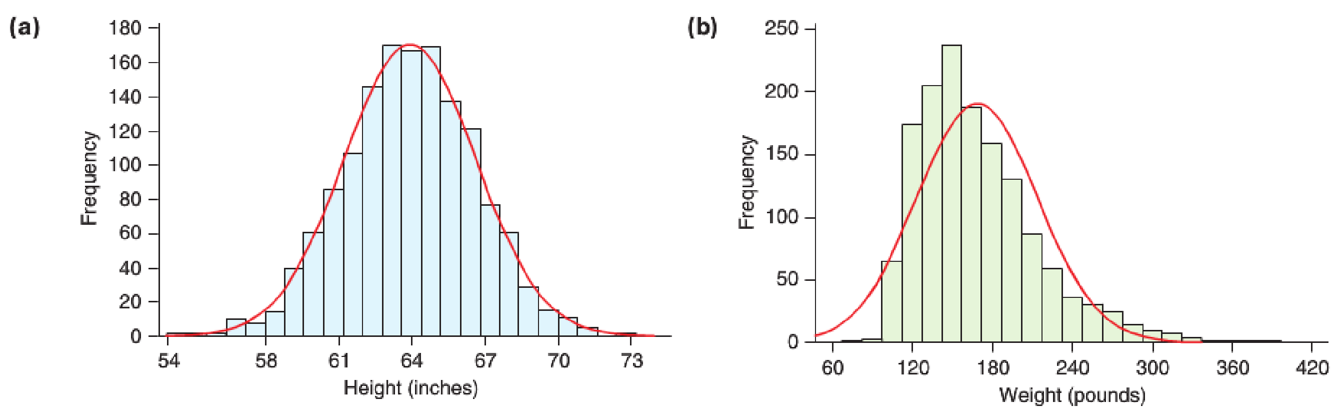
b. The interquartile range is Q3 minus Q1. Find the interquartile range for the test scores.

The interquartile range is ______.
(Round to the nearest integer as needed.)

When to use a Normal model for collected data?

Make the histogram and if the data is unimodal and symmetric then it is appropriate to use the Normal model.

(a) Example of a histogram that indicates that the Normal model can be used.
(b) Example of a histogram that indicates that the Normal model should **not be used (skewed data)**.



▲ **FIGURE 6.27** **(a)** A histogram of data from a large sample of adult women in the United States drawn at random from the National Health and Nutrition Examination Survey. The red curve is the Normal curve, which fits the shape of the histogram very well, indicating that the Normal model would be appropriate for these data. **(b)** A histogram of weights for the same women. Here the Normal model is a bad fit to the data.

6.3 The Binomial Model – model for the special discrete probability distributions

As opposed to normal distributions where the mean and SD are given and we use Normal Calculator to find probability, in some discrete distributions we can use Binomial Calculator to compute probabilities.

In these cases, we are given n (number of trials) and p (probability of each success) and that is eathe siest way to recognize them.

With the increased n **these distributions also become bell-shaped and their mean is getting closer to p.**

The binomial probability model is used for random experiments that satisfy the following conditions:

1. The experiment consists of a **fixed number of trials**. (n)
2. There must be **two outcomes in each trial** (two complementary outcomes)
3. The **probability of success in each trial is the same**. (p)
4. The trials are **independent**.

Example:
Random experiment: Count number of heads in the experiment when we flip a coin 3 times.
Can the binomial probability model be used?

1. the number of trials is 3 ($n = 3$)
2. in each trial we get either heads or tails
3. the probability of heads ($p = 0.5$)
4. the trials are independent

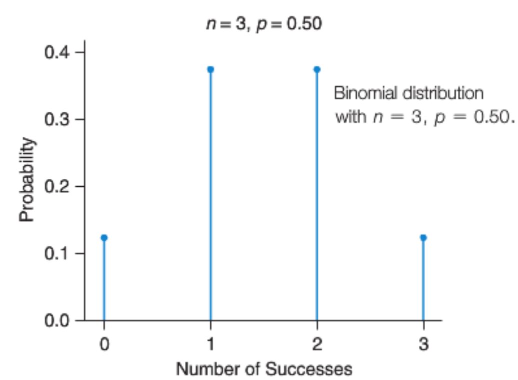
Changeable parts of a binomial probability model (binomial distribution) are:

n and p

Observe: Conditions for probability distribution must hold:

- The probability of each outcome must be in [0, 1]
- The sum of all probabilities must be 1

In this case, the probabilities are all below 0.40



If n is increased we get more possible outcomes and still:

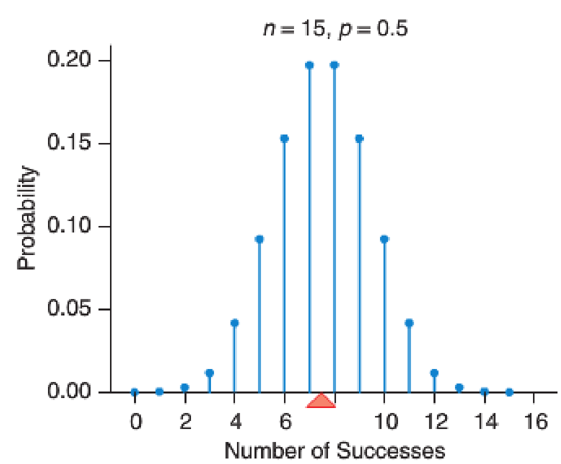
- The probability of each outcome must be in [0, 1]
- The sum of all probabilities must be 1

In this case, the probabilities on the are all below 0.20 (this makes sense because there are more outcomes and the sum still must be 1).

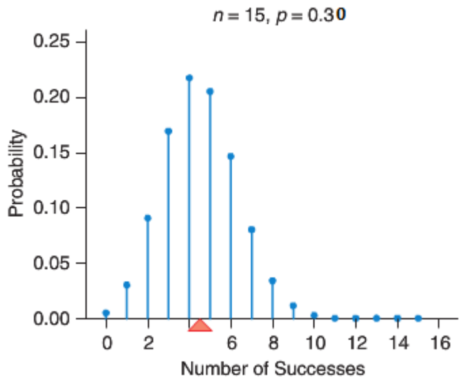
What is the effect of p and ?

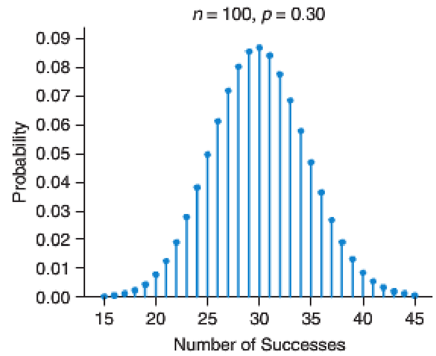
The distribution is symmetrical around np

$$np = 15(0.5) = 7.5$$



Example: $p \neq 0.5$ (in this case p = 0.30) the distribution may be skewed for smaller n (such as n=15) BUT as n **increases** (such as n=100) the distribution becomes symmetrical around the value: $p(n)$ Example 0.30(15)=4.5 or 0.3(100)=30





Observe that probability values became smaller in the second graph (look at y-axis).

Calculate the probability using a binomial model

The binomial distribution is determined by n (number of trials) and p (probability of each success).

Now we can answer the question: **What is the probability** of getting exactly **X successes** in the binomial distribution with n trials and probability p in each success?

Math shorthand for such probability is: $\mathbf{b(n, p, X)}$ where: b indicates binomial model

n is the number of trials

p is the probability of each success

X are values that we are interested in their probability

We can compute the probability of exact number X of successes, we can compute "more than X successes", "more or equal than X successes", "less than X successes"…

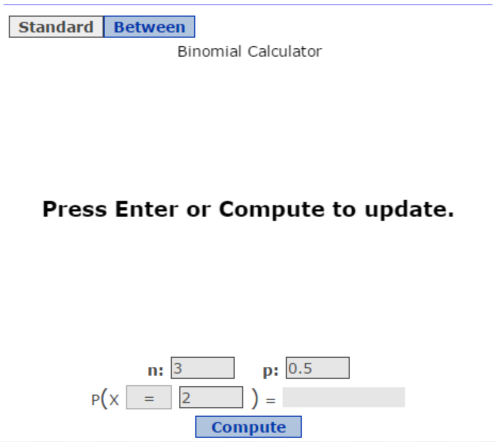
We will use StatCrunch to compute such probabilities.

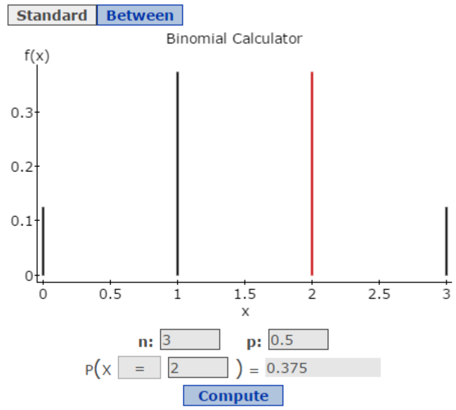
Example:
Random experiment: Count number of heads in the experiment when we flip a coin 3 times.
Compute the probability of getting exactly 2 heads.

a. First, determine if the binomial model can be used for this random experiment.
We earlier verified that binomial model can be used (checked all 4 conditions).

b. If the experiment can be modeled by the binomial model then we use StatCrunch to compute the probability of getting 2 heads: $\mathbf{b(n, p, X) = b(3,\ 0.5,\ 2)}$ (n=3 p=0.5 and X=2).

Work: In **StatCrunch go to: Stat -> Calculators -> Binomial**
Enter given values for $\mathbf{n}$, $\mathbf{p}$, and $\mathbf{X}$ and select Compute Write: $\mathbf{b}(3, 0.5, 2) = 0.375 = 37.5\%$





Extra practice: Now compute the probability of:

a. More than 2 heads

b. 2 or fewer heads

a. Set the operation to > in StatCrunch. Answer: $\mathbf{b}(3, 0.5, X > 2) = 0.125$

b. Set the operation to ≤ in StatCrunch. Answer: $\mathbf{b}(3, 0.5, X \leq 2) = 0.875$

"X or fewer" probabilities are called **cumulative probabilities.**

A binomial probability experiment is conducted with the given parameters. Compute the probability of x successes in the n independent trials of the experiment. $n = 9,\ p = 0.6,\ x \leq 3$

Work: In StatCrunch go to: Stat -> Calculators -> Binomial

Historically, about 90% of MBAs (people who have earned a master's degree in business administration) from a certain university find a job within three months of graduation. Assume for the sake of simplicity that whether a graduate finds a job in three months is independent of whether any of the other graduates find jobs. Complete parts a through c below.

a. Find the probability that at most 5 (this means 5 or fewer) out of 10 will find a job in their chosen field.

b. Find the probability that at least 6 (this means 6 or more) out of 10 will find a job in their chosen field.

c. Find the probability that anywhere from 5 to 7 out of 10 will find a job in their chosen field. The 5 to 7 is inclusive—that is, it includes the values for 5 and 7.

Work: In StatCrunch go to: Stat -> Calculators -> Binomial

Formulas for the mean and SD for the binomial distribution in terms of a number of trials (n) and the probability of success (p).

$\mu = np$ $\sigma = \sqrt{np(1-p)}$ the distribution is $b(np, \sqrt{np(1-p)})$

The mean of any probability distribution is also called the expected value of a probability distribution.

Extra reference for binomial vs. proportions: http://www.stat.yale.edu/Courses/1997-98/101/binom.htm

According to a study in 2010, 15% of women in a certain country have ended their childbearing years without having children (In the 1970s, this number was 10%). Complete parts a through c below.

a. If we randomly select 400 women, how many would we expect to have had no children? Give or take how many?

The expected number of women who had no children is ______, give or take ______.
(Round to the nearest integers as needed.)

b. Give the range of likely values from 1 standard deviation below the mean to 1 standard deviation above the mean.

You should expect from ______ to ______ women to have no children.
(Use ascending order. Round to the nearest integers as needed.)

c. If you found that 70 out of 400 randomly sampled women ended their childbearing years without having children, would you be surprised? Why or why not?

- **A.** No, because 70 is close to the mean (within 1 SD).
- **B.** Yes, because 70 is far from the mean (not within 1 SD).
- **C.** No, because 70 is far from the mean (not within 1 SD).
- **D.** Yes, because 70 is close to the mean (within 1 SD).

STANDARD NORMAL DISTRIBUTION Table

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997